$\qquad$

## Prerequisite Skills

## Key Features of Quadratic Functions

Consider the function $y=x^{2}-7 x+12$.

1. a) Create a table of values and graph the function.
b) Describe the shape of the graph. What type of function is this?
c) Find the first and second differences.

Explain how this supports your answer to part b).
2. Refer to the function in question 1.

Identify the
a) $x$-intercepts
b) $y$-intercept
c) direction of opening
d) equation of the axis of symmetry
e) coordinates of the vertex
3. Sketch the graph of a parabola whose vertex is at $(1,-4)$ and whose $x$-intercepts are -1 and 3 .

## Expand and Simplify Algebraic Expressions

4. Use algebra tiles to expand $3(2 x-1)$.
5. Expand, using the distributive property.
a) $4(x+3)$
b) $-3(y-5)$
c) $x(x+7)$
d) $2 m(3 m-4)$
e) $6\left(x^{2}+2 x-1\right)$
f) $-\frac{1}{2}\left(2 k^{2}-4 k+6\right)$
6. a) Use algebra tiles to model each expression.
i) $x+1$
ii) $x+3$
b) Use algebra tiles to build a rectangle with length $x+1$ and width $x+3$.
c) What is the area of the rectangle?
7. Expand and simplify.
a) $(x+3)(x+5)$
b) $(u-2)(u+3)$
c) $(k+5)(k-5)$
d) $(2 p-8)(p+3)$
e) $(2 g+5)^{2}$
f) $(3 d-1)^{2}$
8. Expand and simplify.
a) $2(x+2)+5(x-4)$
b) $v(v-2)-2 v(4 v+3)$
c) $(y+5)(y-2)+(y-3)(y+1)$
d) $(2 n-1)^{2}+2(n+3)$

## Factors of Polynomials

9. a) Select algebra tiles to represent the polynomial $3 x+6$.
b) Arrange the tiles to form a rectangle.
c) What are the expressions for the length and width of the rectangle?
d) Explain how this model illustrates how to factor $3 x+6$.
10. Use algebra tiles to show the factors of each polynomial.
a) $2 x+6$
b) $3 x+12$
11. Factor out the greatest common factor.
a) $2 x+16$
b) $6 h-36$
c) $c^{2}+3 c$
d) $3 y^{2}-9 y$
e) $3 m^{2}+9 m+6$
f) $2 r^{2}+4 r-8$
12. Factor each trinomial.
a) $x^{2}+3 x+2$
b) $m^{2}-9 m+8$
c) $y^{2}+6 y+9$
d) $p^{2}+p-6$
$\qquad$

## Section 2.1 Quadratic Functions: Exploring Forms

Use the following quadratic functions to answer questions $1-5$.

A $y=x^{2}+6 x+8$
B $y=(x-3)^{2}+7$
C $y=(x-3)(x+5)$
D $y=-2(x+4)^{2}-1$
E $y=-(x+1)(x+5)$
F $y=-x^{2}+x-3$

1. Identify the algebraic form used to express each function: standard form, factored form, or vertex form.
2. For those functions in the standard form, identify
a) the values of $a, b$, and $c$
b) the direction of opening
c) whether the vertex is a maximum or a minimum
d) the $y$-intercept
3. For those functions in the factored form, identify
a) the direction of opening
b) the $x$-intercepts
c) the coordinates of the vertex
d) whether the vertex is a maximum or a minimum
e) the axis of symmetry
f) the $y$-intercept
4. For those in vertex form, identify
a) the direction of opening
b) the coordinates of the vertex
c) whether the vertex is a maximum or a minimum
d) the axis of symmetry
e) the $y$-intercept
5. a) Sketch a graph of each function, without graphing technology.
b) Use graphing technology to help you identify the intervals for which each function is
i) positive or negative
ii) increasing or decreasing
6. Determine an equation for each quadratic function.
a) The $y$-intercept is 2 and the vertex is $(4,-2)$.
b) The $x$-intercepts are 3 and 7 and the minimum value is -8 .
c) The vertex is $(-2,4)$ and one $x$-intercept is 0 .
7. This graph shows the height-distance relationship of a baseball hit for a home run.

a) What is the approximate maximum height reached?
b) What is the approximate distance of the home run?
c) Write a quadratic function that approximates the trajectory of the home run. (It will not be the same as the graph because the ball falls faster than it rises.)
$\qquad$

## Section 2.2 Achievement Check Rubric

| Categories | Level 1 | Level 2 | Level 3 | Level 4 |
| :---: | :---: | :---: | :---: | :---: |
| Knowledge and Understanding <br> - Graphs the two quadratic equations and chooses the maximum point. <br> - Compares the areas to find the largest one. | Demonstrates limited understanding of solving a quadratic equation graphically and cannot find the required areas. | Demonstrates some understanding of solving a quadratic equation graphically or finds the required areas. | Demonstrates considerable understanding of solving a quadratic equation graphically and finds the required areas. | Demonstrates thorough understanding of solving a quadratic equation graphically and finds the required areas accurately. |
| Thinking <br> - Prepares a plan to complete the task. <br> - Carries out the plan. | Needs extensive assistance to begin organizing a plan and needs clearly laid out steps to follow. | Needs some assistance to begin organizing a plan and needs some steps to follow. | Needs minimal assistance to organize and implement an effective strategy. | Needs no assistance to organize and implement an effective strategy. |
| Communication <br> - Correct use of mathematical language. <br> - Clear explanations and full justifications. | Maintains the correct units in some parts of the solution. Does not clearly explain or justify solution. | Maintains the correct units throughout most part of the solution. Explains and justifies solution somewhat. | Maintains the correct units throughout the solution. Explains and justifies solution fully. | Maintains the correct units throughout the solution. Explains, justifies, and shows insight into the complexities of the solution. |
| Application <br> - Illustrates the problem with a diagram. <br> - Models the problem with the two quadratic equations. | Interprets the information ineffectively with an unlabelled diagram and requires significant assistance to find the quadratic equations. | Interprets the information somewhat effectively with a diagram or requires some assistance to find the quadratic equations. | Interprets the information with considerable effectiveness with labelled diagrams and requires little assistance to find the quadratic equations | Interprets the information with a high degree of effectiveness with accurately labelled diagrams and finds the two correct quadratic equations. |

$\qquad$

## Section 2.3 Factor Quadratic Expressions of the Form $a x^{2}+b x+c$

1. Factor each trinomial, using algebra tiles, if possible. If not possible, write not factorable.
a) $x^{2}+4 x+3$
b) $b^{2}+6 b+8$
c) $p^{2}+6 p+5$
d) $v^{2}+3 v+5$
e) $w^{2}+8 w+12$
2. Factor each trinomial, if possible. If not possible, write not factorable.
a) $y^{2}-13 y+30$
b) $k^{2}+11 k+24$
c) $c^{2}+4 c+7$
d) $m^{2}+2 m-48$
e) $a^{2}-7 a+12$
3. Factor each trinomial, if possible. If not possible, write not factorable. Look for common factors first.
a) $2 y^{2}-18 y-20$
b) $3 m^{2}+18 m+15$
c) $2 q^{2}+4 q+16$
d) $5 n^{2}+10 n-15$
e) $3 d^{2}-3 d-36$
4. a) Create a trinomial that can be factored into two binomials.
b) Factor the trinomial.
c) Illustrate how the polynomial is related to its factors using algebra tiles. Draw a diagram of your model.
5. a) Create a trinomial that cannot be factored.
b) Use algebra tiles or algebraic reasoning to explain why it cannot be factored.
6. Factor each trinomial, using algebra tiles, if possible. If not possible, write not factorable.
a) $2 x^{2}+7 x+3$
b) $3 k^{2}+10 k+3$
c) $2 y^{2}+7 y+5$
d) $4 j^{2}+j+3$
e) $4 b^{2}+8 b+3$
7. Factor each trinomial, if possible. If not possible, write not factorable.
a) $3 e^{2}+10 e+8$
b) $2 g^{2}+9 g+9$
c) $2 k^{2}-9 k-5$
d) $9 m^{2}-9 m+9$
e) $12 p^{2}-23 p-2$
8. Factor each trinomial, if possible. If not possible, write not factorable. Look for common factors first.
a) $2 w^{2}+10 w+12$
b) $4 w^{2}+20 w+24$
c) $3 x^{2}+3 x-6$
d) $2 m^{2}+4 m-30$
e) $6 a^{2}-6 a-12$
9. a) Create a trinomial that can be factored into 2 binomials for which $a \neq 1$, where $a$ is the coefficient of the quadratic term.
b) Explain your method using words and diagrams.
10. Which do you think are more common: trinomials that can be factored or trinomials that cannot be factored? Justify your answer with mathematical reasoning.
$\qquad$

## Section 2.4 Select and Apply Factoring Strategies

1. Factor each perfect square trinomial.
a) $x^{2}+10 x+25$
b) $k^{2}-12 k+36$
c) $4 v^{2}+28 v+49$
d) $100 h^{2}+20 h+1$
2. Choose one of the expressions in question 1.
a) Check your result by expanding and simplifying the product of the factors.
b) Sketch an algebra-tile model that relates the trinomial to its factors.
3. Factor each difference of squares.
a) $x^{2}-64$
b) $p^{2}-121$
c) $4 w^{2}-9$
d) $36 m^{2}-169$
4. Choose one of the expressions in question 3.

Check your result by expanding and simplifying the product of the factors.
5. Factor fully, if possible. If not possible, write not factorable. Check for common factors first.
a) $2 x^{2}+12 x+18$
b) $3 w^{2}-48$
c) $k^{4}-16$
d) $n^{3}-9 n$
e) $8 v^{2}+8 v+2$
f) $3 p^{3}-30 p^{2}+75 p$
g) $9 b^{2}-25$
h) $3 y^{2}-12$
i) $9 c^{2}+12 c+4$
j) $x^{4}-625$
k) $7 y^{2}+21 y+35$
l) $6 d^{2}-7 d-20$
m) $9 x^{2}-15 x-6$
n) $5 f^{2}-70 f+245$
о) $12 w^{2}-5 w+6$
р) $7 b^{2}-63$
q) $11 q^{2}+11 q-220$
r) $15 h^{2}-2 h-8$
s) $20 m^{3}-45 m$
t) $9 a^{2}+36 a+36$
6. Choose two expressions in question 5. Check your result by expanding and simplifying the product of the factors.
7. a) Write the quadratic function $y=x^{2}+10 x+25$ in factored form.
b) How many zeros does this function have? Explain how you can tell.
8. a) Write the quadratic function $y=x^{2}-16$ in factored form.
b) Identify the axis of symmetry for the graph of the function. Explain how you can tell.
9. a) Give an example of a polynomial that can be factored by factoring out the greatest common factor and factoring a difference of squares.
b) Prove that your example works.
10. a) Give an example of a polynomial that can be factored by applying two strategies of your choice.
b) Identify the strategies and prove that your example works.
$\qquad$

## Section 2.5 Solve Quadratic Equations by <br> Factoring

1. Find the roots of each equation. Verify your answers.
a) $x(x-5)=0$
b) $(y+2)(y-9)=0$
c) $(k+9)(2 k+3)=0$
d) $(3 m-1)(2 m+5)=0$
e) $(3 g-12)(11 g+22)=0$
f) $(3 t-3)(t+5)=0$
2. Find the roots of each equation. Verify your answers.
a) $x^{2}+6 x+9=0$
b) $w^{2}-16=0$
c) $g^{2}-5 g+6=0$
d) $u^{2}-6 u-7=0$
e) $z^{2}+21 z+20=0$
f) $2 k^{2}-50=0$
g) $n^{2}-7 n-8=0$
h) $2 v^{2}+3 v-2=0$
i) $2 p^{2}-30 p+100=0$
j) $4 b^{2}-25=0$
k) $2 t^{2}+21 t+10=0$
l) $3 y^{2}-13 y+4=0$
m) $9 c^{2}+12 c+4=0$
n) $5 x^{2}-25 x+20=0$
о) $7 y^{2}-28 y-35=0$
р) $6 d^{2}-7 d+2=0$
q) $9 x^{2}-15 x-6=0$
r) $5 f^{2}-5 f-100=0$
s) $12 w^{2}-8 w+1=0$
t) $12 d^{2}-10 d+2=0$
3. A ball is tossed from a building. Its height as a function of time is given by
$h(t)=-5 t^{2}+15 t+50$, where $h$ is the height of the ball above ground, in metres, $t$ seconds after being tossed.
a) Find the zeros of the function and explain their significance. Reject any inadmissible solutions.
b) Graph the function. Which parts of the graph have no meaning in this situation?
4. a) Create a quadratic function in standard form that has zeros at 2 and -3 .
b) Explain how you produced this function.
c) Is your function the only possible correct answer to part a)? Explain.
5. The sum of the squares of two consecutive whole numbers is 85 . What are the numbers?
6. A rectangle is 3 m longer than it is wide. Its area is $154 \mathrm{~cm}^{2}$. What are the dimensions of the rectangle?
7. A soccer ball is kicked so that its height, $h$, after $t$ seconds is modelled by the function $h(t)=20 t-5 t^{2}$.
a) When will the height of the ball be 15 m ? Explain why there are two answers.
b) What is the maximum height of the ball?
c) How long will the ball be in the air?
8. The sum of the first $n$ natural numbers is given by the function $\frac{1}{2} n(n+1)$.

$$
1+2+3+\ldots+n=\frac{1}{2} n(n+1)
$$

If the sum is 55 , what is the value of $n$ ?
9. A tennis court is 13 m longer than it is wide. Its area is $264 \mathrm{~m}^{2}$. What are the lengths of its sides?
$\qquad$

## Chapter 2 Review

### 2.1 Quadratic Functions: Exploring Forms, pages 64-75

1. Consider the quadratic function $f(x)=(x+6)(x-4)$.
a) In which algebraic form is this function expressed? Does the parabola open upward or downward?
b) What are the $x$-intercepts?
c) Find the coordinates of the vertex. Is the vertex a maximum or a minimum?
Explain.
d) What is the axis of symmetry?
e) What is the $y$-intercept?
f) Graph the function.
g) Identify the intervals for which the function is
i) positive or negative
ii) increasing or decreasing
2. Consider the quadratic function

$$
y=3(x+1)^{2}+2 .
$$

a) In which algebraic form is this function expressed? Does the parabola open upward or downward?
b) What are the coordinates of the vertex? Is the vertex a maximum or a minimum? Explain.
c) What is the axis of symmetry?
d) What is the $y$-intercept?
e) Graph the function.
f) Identify the intervals for which the function is increasing or decreasing.
3. Consider the quadratic function $f(x)=x^{2}-6 x+10$.
a) In which algebraic form is this function expressed?
b) Identify the values of $a, b$, and $c$. Does this parabola open upward or downward?
c) What is the $y$-intercept?
d) Create a table of values and graph the function. Use enough values to get a good idea of the shape of the graph.
e) Estimate the value of the
i) $x$-intercepts
ii) vertex
f) Is the vertex a maximum or a minimum? Explain.
4. a) Write a quadratic function with the vertex at $(2,3)$ and $x$-intercepts of -1 and 5 .
b) Sketch a graph of your function.

### 2.2 Quadratic Functions: Comparing Forms, pages 76-85

5. For each quadratic function
a) $y=(x+3)(x-4)$
b) $g(x)=(x-1)^{2}+5$
i) express the function in standard form
ii) identify the values of $a, b$, and $c$
iii) identify the $y$-intercept
6. Expand and simplify each expression.
a) $5(x-3)-2(3 x+1)$
b) $3 y(y+2)+2(4 y-6)$
c) $(2 a+1)(a-5)+(a-6)(2 a-3)$
d) $(3 b-2)^{2}-(2 b-1)^{2}$
e) $-5(p+1)(p-1)+(2 p-3)$
f) $(q+2)^{2}-(q-2)^{2}$
7. a) Create a quadratic function expressed in factored form.
b) Write the function in standard form.
c) Identify the values of $a, b$, and $c$.
d) Identify the $y$-intercept.
8. a) Create a quadratic function expressed in vertex form.
b) Write the function in standard form.
c) Identify the values of $a, b$, and $c$.
d) Identify the $y$-intercept.

Name: $\qquad$ Date: $\qquad$
2.3 Factor Quadratic Expressions of the Form $a x^{2}+b x+c$, pages 88-97
9. Factor each expression. Use algebra tiles or diagrams to illustrate.
a) $x^{2}+9 x+8$
b) $3 x^{2}+10 x+3$
10. Factor, if possible. If not possible, write not factorable, and explain why.
a) $y^{2}+7 y-18$
b) $k^{2}-5 k-14$
c) $3 v^{2}+4 v+1$
d) $4 n^{2}-4 n-3$
e) $2 r^{2}-2 r+3$
f) $8 x^{2}-10 x-3$
11. Fully factor each expression. Look for common factors first.
a) $2 x^{2}-6 x-20$
b) $3 y^{3}+5 y^{2}-2 y$
12. Write each quadratic function in factored form. Then, identify the $x$-intercepts.
a) $f(x)=x^{2}+9 x-10$
b) $y=2 x^{2}-7 x+3$
13. a) Create a rectangle using algebra tiles that represent $x^{2}, x$, and 1 .
b) Write the corresponding polynomial in standard and factored form.

### 2.4 Select and Apply Factoring Strategies, pages 98-107

14. a) Identify the factors of $x^{2}+8 x+16$.
b) Explain why this polynomial is called a perfect square trinomial, using words, diagrams, and/or algebraic reasoning.
15. Create a perfect square trinomial. Write the expression as an expanded polynomial and as a product of factors.
16. Factor each expression.
a) $x^{2}-18 x+81$
b) $4 m^{2}+12 m+9$
c) $n^{2}-25$
d) $9 v^{2}-100$
17. Fully factor each expression. Look for common factors first.
a) $3 w^{2}-108$
b) $4 g^{2}+60 g+176$
c) $12 n^{2}+8 n-20$
d) $2 k^{3}-8 k^{2}+8 k$

### 2.5 Solve Quadratic Equations by Factoring, pages 108-113

18. Find the roots of each equation.
a) $x^{2}-14 x+24=0$
b) $2 p^{2}+p-3=0$
c) $3 q^{2}+16 q+5=0$
d) $4 m^{2}-1=0$
e) $5 r^{2}-11 r+2=0$
f) $4 n^{2}-2 n-2=0$
19. The sum of two whole numbers is 23 and the sum of their squares is 277 . What are the numbers?
20. A square is surrounded by a sidewalk that is $1-\mathrm{m}$ wide. The area of the square is $1 \mathrm{~m}^{2}$ greater than twice the area of the sidewalk. What are the dimensions of the square?
21. The arms of a right triangle have lengths that differ by 7 cm . The area is $30 \mathrm{~cm}^{2}$. What is the length of the hypotenuse?
22. The lengths of the parallel sides of a trapezoid differ by 10 cm . The height of the trapezoid is the same length as the short parallel side. The area of the trapezoid is $84 \mathrm{~cm}^{2}$. What are the lengths of the parallel sides of the trapezoid?
$\qquad$

## Chapter 2 Practice Test

## For questions 1 to 5, select the best answer.

1. Which statement is true for the function $f(x)=(x-2)^{2}-3$ ?
A The $x$-intercepts are 2 and -3 .
B The $x$-intercepts are -2 and -3 .
C The vertex is at $(2,-3)$.
D The vertex is at $(-2,-3)$.
2. Which statement is true for the function $y=(x+1)(x+5)$ ?
A The $x$-intercepts are 1 and 5 .
B The $x$-intercepts are -1 and -5 .
C The vertex is at $(-1,5)$.
D The axis of symmetry is $x=-1$.
3. Which statement is true for the function $f(x)=x^{2}-3 x-6$ ?
A The vertex is at $(3,-6)$.
B The $y$-intercept is -3 and the parabola opens upward.
$\mathbf{C}$ The $y$-intercept is -6 and the parabola opens upward.
D The $y$-intercept is -6 and the parabola opens downward.
4. Which polynomial and factors does this model represent?


$$
\begin{aligned}
& \text { A } x^{2}+9 x+14=(x+9)(x+14) \\
& \text { B } x^{2}+9 x+14=(x+4)(x+5) \\
& \text { C } x^{2}+9 x+14=(x+7)(x+2) \\
& \text { D none of the above }
\end{aligned}
$$

5. Which is the factored form of $4 x^{2}+1$ ?

A $(2 x-1)(2 x+1)$
B $(2 x+1)^{2}$
C $4 x(x+1)$
D The expression is not factorable.
6. a) Draw an algebra tile area model for the expression $4 x^{2}+7 x+3$
b) Identify the factors of the polynomial.
7. Write each function in standard form. Then, identify the $y$-intercept.
a) $f(x)=4(x-1)(x-5)$
b) $g(x)=-2(x+6)^{2}-3$
8. Fully factor each expression, if possible. If it is not possible, write not factorable and explain why.
a) $25 k^{2}-100$
b) $n^{2}+14 n+49$
c) $t^{2}-4 t-45$
d) $2 x^{2}-20 x+50$
e) $z^{2}-2 z+2$
f) $2 y^{3}-98 y$
9. a) In which form is the function $f(x)=-(x-3)(x+7)$ expressed? Does this parabola open upward or downward?
b) What are the $x$-intercepts?
c) Find the coordinates of the vertex. Is it a maximum or a minimum? Explain.
d) What is the axis of symmetry?
e) What is the $y$-intercept?
f) Graph the function.
g) Identify the intervals for which the function is
i) positive or negative
ii) increasing or decreasing
10. The flight path of a firework is modelled by the function $h(t)=-5(t-5)^{2}+127$, where $h$ is the height, in metres, $t$ seconds after being fired.
a) Graph the function.
b) What is the maximum height reached by the firework?
c) How high above ground was the firework fired from?
d) What is the hang time of the firework?
$\qquad$

## Chapter 2 Test

For questions 1 to 5, choose the best answer.

1. Which statement is true for the function $y=(x-3)(x+1)$ ?
A The $x$-intercepts are 3 and -1 .
B The $x$-intercepts are -3 and 1 .
C The vertex is $(1,-3)$.
D The axis of symmetry is $x=1$.
2. Which statement is true for the function $f(x)=-x^{2}+2 x+3$ ?
A The vertex is at $(2,3)$.
B The vertex is at $(-1,4)$.
C The $y$-intercept is -3 and the parabola opens downward.
D The $y$-intercept is 3 and the parabola opens upward.
3. Which statement is true for the function $f(x)=(x+2)^{2}+5$ ?
A The $x$-intercepts are 2 and -5 .
B The $x$-intercepts are -2 and 5 .
C The vertex is at $(2,-5)$.
D The vertex is at $(-2,5)$.
4. Which polynomial and factors does this model represent?


A $2 x^{2}+7 x+6=(2 x+3)(x+2)$
B $2 x^{2}+7 x+6=(2 x+1)(x+6)$
C $x^{2}+7 x+6=(x+2)(x+3)$
D $x^{2}+7 x+6=(x+1)(x+6)$
5. Which is the vertex form of $2 x-x^{2}$ ?
$\mathbf{A}-x(x+2)$
B $x(x-2)$
C $-(x-1)^{2}+1$
D $(x-1)^{2}-1$
6. a) Build or draw an algebra tile area model for the expression $4 x^{2}+4 x+1$.
b) Use the model to identify the factors of the polynomial.
7. Write each quadratic function in factored form. Then, identify the $x$-intercepts.
a) $f(x)=x^{2}-x-12$
b) $g(x)=2 x^{2}+9 x-5$
8. Fully factor each expression, if possible. If it is not possible, write not factorable and explain why.
a) $w^{2}-5 w-6$
b) $4 t^{2}-12 t+9$
c) $27 x^{2}-75$
d) $6 y^{2}-y-15$
e) $2 a^{3}+14 a^{2}+20 a$
f) $b^{2}-b+1$
9. a) In which form is the function $y=3(x-2)^{2}-5$ expressed? Does this parabola open upward or downward?
b) Find the coordinates of the vertex. Is it a maximum or a minimum? Explain.
c) What is the axis of symmetry?
d) What is the $y$-intercept?
e) Graph the function.
f) Identify the intervals for which the function is increasing or decreasing.
10. The path of a football can be modelled by the function $h(t)=-\frac{1}{16} d(d-56)$, where $h$ is the height of the football, in metres, and $d$ is the horizontal distance from the kicker.
a) Graph this relation.
b) How far down field does the football travel before its first bounce?
c) What is the maximum height reached by the football?
d) At what horizontal distance does it reach this height?

## Chapter 2 BLM Answers

## Prerequisite Skills

1. a)

| $x$ | $y$ |
| :---: | ---: |
| 0 | 12 |
| 1 | 6 |
| 2 | 2 |
| 3 | 0 |
| 4 | 0 |
| 5 | 2 |
| 6 | 6 |
| 7 | 12 |


b) parabola; quadratic function


The second differences are constant.
2. a) 3 and 4
b) 12
c) upward
d) $x=3.5$
e) $(3.5,-0.25)$
3.

4. $6 x-3$

5. a) $4 x+12$
b) $-3 y+15$
c) $x^{2}+7 x$
d) $6 m^{2}-8$
e) $6 x^{2}+12 x-6$
f) $-k^{2}+2 k-3$
6. a) i)

ii) $\square \square \square \square$
b)

c) $x^{2}+4 x+3$
7. a) $x^{2}+8 x+15$
b) $u^{2}-u-6$
c) $k^{2}-25$
d) $2 p^{2}-2 p-24$
e) $4 g^{2}+20 g+25$
f) $9 d^{2}-6 d+1$
8. a) $7 x-16$ b) $-7 v^{2}-8 v$ c) $2 y^{2}+y-13$ d) $4 n^{2}-2 n+7$
9. a)

c) $3, x+2$
d) Since the area of the rectangle is $3 x+6$,
$3 x+6=3(x+2)$.
10. a) $2 x+6=2(x+3)$

b) $3 x+12=3(x+4)$

11. a) $2(x+8)$
b) $6(h-6)$
c) $c(c+3)$
d) $3 y(y-3)$
e) $3\left(m^{2}+3 m+2\right)$
f) $2\left(r^{2}+2 r-4\right)$
12. a) $(x+2)(x+1)$
b) $(m-8)(m-1)$
c) $(y+3)^{2}$
d) $(p+3)(p-2)$

## Section 2.1 Quadratic Functions: Exploring Forms

1. $\mathbf{A}, \mathbf{F}$ standard form; $\mathbf{B}, \mathbf{D}$ vertex form; $\mathbf{C}, \mathbf{E}$ factored form
2. A a) $a=1, b=6, c=8$
b) opens upward
c) minimum
d) 8
F a) $a=-1, b=1, c=-3$
b) opens downward
c) maximum
d) -3
3. C a) opens upward
b) 3 and -5
c) $(-1,-16)$
d) minimum
e) $x=-1$
f) -15

E a) opens downward
b) -1 and -5
c) $(-3,4)$
d) maximum
e) $x=-3$
f) -5
4. B a) opens upward
b) $(3,7)$
c) minimum
d) $x=3$
e) 16

D a) opens downward
b) $(4,1)$
d) $x=-4$
e) -33
5. a) A $y=x^{2}+6 x+8$


B $y=(x-3)^{2}+7$


C $y=(x-3)(x+5)$


D $y=-2(x+4)^{2}-1$


E $y=-(x+1)(x+5)$


F $y=-x^{2}+x-3$

b) A i) positive for $x<-4$ and $x>-2$; negative for $-4<x<-2$
ii) increasing for $x>-3$; decreasing for $x<-3$

B i) positive for all $x$ ii) increasing for $x>3$; decreasing for $x<3$
Ci) positive for $x<-5$ and $x>3$; negative for $-5<x<3$
ii) increasing for $x>-1$; decreasing for $x<-1$

D i) negative for all $x$ ii) increasing for $x<-4$; decreasing for $x>-4$
E i) negative for $x<-5$ and $x>-1$; positive for $-5<x<1$
ii) increasing for $x<-3$; decreasing for $x>-3$
$F$ i) negative for all $x$ ii) increasing for $x<0.5$; decreasing for $x>0.5$
6. a) $y=\frac{1}{4}(x-4)^{2}-2$
b) $y=2(x-3)(x-7)$
c) $y=-(x+2)^{2}+4$
7. Answers may vary.
a) approximately 25 m
b) approximately 130 m
c) $y=\frac{1}{153} x(x-130)$

## Section 2.2 Quadratic Functions: Comparing Forms

1. a) $3 x-27$ b) $-4 b^{2}-8 b$ c) $12 k^{2}-18 k-6$ d) $12 q^{2}-15$
2. a) $x^{2}+3 x-10$
b) $12 g^{2}-5 g-2$ c) $4 w^{2}-9$
d) $16 n^{2}-24 n+9$
e) $9 g^{2}+6 g+1$
f) $16 f^{2}-40 f+25$
g) $y^{2}+5 y-9$
h) $-3 v^{2}+26 v-49$
3. a) i) opens upward
$\begin{array}{ll}\text { ii) } 2 \text { and }-4 & \text { iii) }(-1,-9)\end{array}$
iv) minimum
v) $x=-1$
b) $y=x^{2}+2 x-8$
c) i) $a=1, b=2, c=-8$
ii) -8
d)

4. a) i) opens downward
ii) -3 and -7
iii) $(-5,8)$ iv) maximum
v) $x=-5$
b) $y=-2 x^{2}-20 x-42$
c) $a=-2, b=-20, c=-42$
d)

5. a) i) opens downward
ii) $(3,5)$ iii) maximum
iv) $x=3$
b) $y=-x^{2}+6 x-4$
c) i) $a=-1, b=6, c=-4$ ii) -4
d)

6. a) i) opens upward ii) $(-2,-4)$
iii) minimum
iv) $x=-2$
b) $y=3 x^{2}+12 x+8$
c) i) $a=3, b=12, c=8$
ii) 8
d)

7. a) factored form; it is equivalent to $p=-(x-0)(x-12)$
b) 0 and 12
c) $p=-x^{2}+12 x$
d) $(6,36)$; maximum
e)

8. a) 36
b) 20.25
c) 0.0625
9. a) $x^{2}+(x-6)^{2}$, or $2 x^{2}-12 x+36$
b) $3,-3$
10. 25 m by 50 m

Section 2.3 Factor Quadratic Expressions of the Form $a x^{2}+b x+c$

1. a) $(x+3)(x+1)$
b) $(b+2)(b+4)$
c) $(p+5)(p+1)$
d) not factorable
e) $(w+2)(w+6)$
2. a) $(y-15)(y+2)$
b) $(k+3)(k+8)$
c) not factorable
d) $(m-6)(m+8)$
e) $(a-3)(a-4)$
3. a) $2(y-10)(y+1)$ b) $3(m+5)(m+1)$
c) $2\left(q^{2}+2 q+8\right) ; q^{2}+2 q+8$ is not factorable
d) $5(n+3)(n-1)$ e) $3(d-4)(d+3)$
4. Answers may vary. For example,
a) $x^{2}+3 x+2$
b) $(x+1)(x+2)$
c)

5. Answers may vary. For example,
a) $x^{2}+x+2$
b) If $=x^{2}+x+2=(x+a)(x+b)$, then $a+b=1$ and $a \times b=2$. $a$ and $b$ must be both positive ( 1 and 2) or both negative $(-1$ and -2$)$ for their product to be 2 . Since $1+2 \neq 1$ and $(-1)+(-2) \neq 1$, the trinomial $x^{2}+x+2$ is not factorable.
6. a) $(x+3)(2 x+1)$ b) $(k+3)(3 k+1)$ c) $(2 y+5)(y+1)$
d) not factorable e) $(2 b+3)(2 b+1)$
7. a) $(3 e+4)(e+2)$ b) $(2 g+3)(g+3)$ c) $(2 k-1)(k+5)$
d) $9\left(m^{2}-m+1\right) ; m^{2}-m+1$ is not factorable
e) $(12 p+1)(p-2)$
8. а) $2(w+2)(w+3)$
b) $4(w+2)(w+3)$
c) $3(x+3)(x-2)$
d) $2(m+5)(m-3)$
e) $6(a+1)(a-2)$
9. Answers may vary. For example,
a) $2 x^{2}+3 x+1$
b) Multiply two binomials $x+1$ and $2 x+1$, and simplify. $(x+1)(2 x+1)=2 x^{2}+3 x+1$

10. Answers may vary. For example, trinomials that cannot be factored may be more common since it is unlikely that three coefficients chosen at random will make a factorable trinomial.

## Section 2.4 Select and Apply Factoring Strategies

1. a) $(x+5)^{2}$
b) $(k-6)^{2}$
c) $(2 v+7)^{2}$
d) $(10 h+1)^{2}$
2. Answers may vary. For example,
a) $(x+5)(x+5)=x^{2}+5 x+5 x+25=x^{2}+10 x+25$
b)

3. a) $(x+8)(x-8)$
b) $(p+11)(p-11)$
c) $(2 w+3)(2 w-3)$
d) $(6 m+13)(6 m-13)$
4. Answers may vary. For example,
a) $(x+8)(x-8)=x^{2}+8 x-8 x-64=x^{2}-64$
5. a) $2(x+3)^{2}$
b) $3(w+4)(w-4)$
c) $\left(k^{2}+4\right)(k+2)(k-2)$
d) $n(n+3)(n-3)$
e) $2(2 v+1)^{2}$
f) $3 p(p-5)^{2}$
g) $(3 b+5)(3 b-5)$
h) $3(y+2)(y-2)$
i) $(3 c+2)^{2}$
j) $\left(x^{2}+25\right)(x+5)(x-5)$
k) $7\left(y^{2}+3 y+5\right) ; y^{2}+3 y+5$ is not factorable
l) $(3 d+4)(2 d-5) \quad$ m) $3(3 x+1)(x-2)$
n) $5(f-7)^{2}$
o) not factorable
р) $7(b+3)(b-3)$
q) $11(q+5)(q-4)$
r) $(3 h+2)(5 h-4)$
s) $5 m(2 m+3)(2 m-3)$
t) $9(a+2)^{2}$
6. Answers may vary. For example,
a) $2(x+3)(x+3)=2\left(x^{2}+3 x+3 x+9\right)$

$$
\begin{aligned}
& =2\left(x^{2}+6 x+9\right) \\
& =2 x^{2}+12 x+18
\end{aligned}
$$

b) $3\left(w^{2}+4 w-4 w-16\right)=3\left(w^{2}-16\right)$

$$
=3 w^{2}-48
$$

7. a) $y=(x+5)^{2}$
b) one; there is only one binomial factor $x+5$ that can be zero
8. a) $y=(x+4)(x-4)$
b) the $y$-axis, because it is midway between $x=-4$ and $x=4$
9. Answers may vary. For example,
$\begin{array}{ll}\text { a) } 3 x^{2}-12 & \text { b) } 3 x^{2}-12=3\left(x^{2}-4\right)=3(x+2)(x-2)\end{array}$
10. Answers may vary. For example,
a) $3 x^{2}-12 x+12$
b) factor out the greatest common factor: $3 x^{2}-12 x+12=3\left(x^{2}-4 x+4\right)$; factor the perfect square trinomial: $3\left(x^{2}-4 x+4\right)=3(x-2)^{2}$

## Section 2.5 Solve Quadratic Equations by Factoring

1. a) 0,5
b) $-2,9$
c) $-9,-1.5$
d) $\frac{1}{3},-2.5$
e) $4,-2$
f) $1,-5$
2. а) -3
b) $4,-4$
c) 2,3
d) $7,-1$
e) $-1,-20$
f) $5,-5$
g) $8,-1$
h) $-2,0.5$
i) 5, 10
j) $2.5,-2.5$
k) $-0.5,-10$
l) $\frac{1}{3}, 4$
m) $-\frac{2}{3}$
п) 1,4
о) $5,-1$
р) $\frac{2}{3}, \frac{1}{2}$
q) $-\frac{1}{3}, 2$
r) $5,-4$
s) $\frac{1}{2}, \frac{1}{6}$
t) $\frac{1}{2}, \frac{1}{3}$
3. a) 5 and $-2 ;-2 \mathrm{~s}$ is inadmissible; the ball lands on the ground after 5 s .
b) The part of the graph for which $t<0$ has no meaning.

4. Answers may vary. For example,
a) $y=x^{2}+x-6$
b) Multiply $(x-2)$ by $(x+3)$.
c) Any multiple of $x^{2}+x-6$, such as $2 x^{2}+2 x-12$, has the same zeros.
5. 6 and 7
6. 11 m by 14 m
7. a) after 1 s and after 3 s ; the ball is at 15 m on its way up in the air and on its way back down
b) $20 \mathrm{~m} \quad$ c) 4 s
8. $n=10$
9. 11 m by 24 m

## Chapter 2 Review

1. a) factored form; opens upward
b) -6 and 4
c) $(-1,-25)$; minimum, because the parabola opens upward
d) $x=-1$
e) -24
f)

g) i) positive for $x<-6$ and $x>4$; negative for $-6<x<4$ ii) increasing for $x>-1$; decreasing for $x<-1$
2. a) vertex form; opens upward
b) $(-1,2)$; minimum, because the parabola opens upward
c) $x=-1 \quad$ d) 5
e)

f) increasing for $x>-1$; decreasing for $x<-1$
3. a) standard form
b) $a=1, b=-6, c=10$; opens upward
c) 10
d)

| $\boldsymbol{x}$ | $\boldsymbol{f}(\boldsymbol{x})$ |
| ---: | ---: |
| -2 | 26 |
| -1 | 17 |
| 0 | 10 |
| 1 | 5 |
| 2 | 2 |
| 3 | 1 |
| 4 | 2 |
| 5 | 5 |
| 6 | 10 |
| 7 | 17 |
| 8 | 26 |

e) i) no $x$-intercepts ii) $(3,1)$
f) minimum, because the parabola opens upward
4. a) $y=-\frac{1}{3}(x-5)(x+1)$
b)

5. a) i) $y=x^{2}-x-12$
ii) $a=1, b=-1, c=-12$ iii) -12
ii) $a=1, b=-2, c=6$
b) i) $y=x^{2}-2 x+6$
iii) 6
6. a) $-x-17$
b) $3 y^{2}+14 y-12$
c) $4 a^{2}-24 a+13$
d) $5 b^{2}-8 b+3$ e) $-5 p^{2}+2 p+2$
f) $8 q$
7. Answers may vary. For example,
a) $y=(x+4)(x-4)$
b) $y=x^{2}-16$
c) $a=1, b=0, c=-16$
d) -16
8. Answers may vary. For example,
a) $y=2(x+5)^{2}-1$
b) $y=2 x^{2}+20 x+49$
c) $a=2, b=20, c=49$
d) 49
9. a) $(x+8)(x+1)$

b) $(3 x+1)(x+3)$

10. a) $(y+9)(y-2)$
b) $(k-7)(k+2)$ c) $(3 v+1)(v+1)$
d) $(2 n-3)(2 n+1)$
e) not factorable f) $(2 x-3)(4 x+1)$
$\begin{array}{ll}\text { 11. a) } 2(x+2)(x-5) & \text { b) } y(3 y-1)(y+2)\end{array}$
12. a) $f(x)=(x+10)(x-1) ;-10$ and 1
b) $y=(2 x-1)(x-3) ; 0.5$ and 3
13. Answers may vary. For example,

b) $2 x^{2}+8 x+6 ;(2 x+2)(x+3)$
14. a) $(x+4)(x+4)$
b) Both factors are the same, so the product is the square of the binomial $x+4$.
15. Answers may vary. For example, $(2 x+1)^{2}=4 x^{2}+4 x+1$
16. a) $(x-9)^{2}$
b) $(2 m+3)^{2}$
c) $(n+5)(n-5)$
d) $(3 v+10)(3 v-10)$
17. a) $3(w+6)(w-6)$
b) $4(g+4)(g+11)$
c) $4(3 n+5)(n-1)$
d) $2 k(k+2)^{2}$
18. a) 2,12
b) $1,-1.5$
c) $-5,-\frac{1}{3}$
d) $\frac{1}{2},-\frac{1}{2}$
e) $2, \frac{1}{5}$
f) $1,-\frac{1}{2}$
19. 14 and 9
20. 9 m by 9 m
21. 13 cm
22.7 cm and 17 cm

## Chapter 2 Practice Test

1. C 2. B 3. C 4. C 5. D
2. a)

b) $(4 x+3)(x+1)$
3. a) $f(x)=4 x^{2}-24 x+20 ; 20$ b) $g(x)=-2 x^{2}-12 x-75 ;-75$
4. a) $25(k-2)(k+2)$
b) $(n+7)^{2}$
c) $(t-9)(t+5)$
d) $2(x-5)^{2}$
e) not factorable; the parabola opens upward and has a minimum of 1 , so it never has a value of 0 ; there are no $x$-intercepts
f) $2 y(y+7)(y-7)$
5. a) factored form; opens downward $\quad$ b) 3 and -7
c) $(-2,25)$; maximum, because the parabola opens downward
d) $x=-2$
e) 21
f)

g) i) positive for $-7<x<3$; negative for $x<-7$ and $x>3$ ii) increasing for $x<-2$; decreasing for $x>-2$
6. a)

b) 127 m c) 2 m d) about 10 s

## Chapter 2 Test

1. D 2. B 3. D 4. A 5. C
2. a)

b) $(4 x+1),(4 x+1)$
3. a) $(x-4)(x+3) ; 4$ and -3 b) $(2 x-1)(x+5) ; \frac{1}{2}$ and -5
$\begin{array}{lll}\text { 8. a) }(w+1)(w-6) & \text { b) }(2 t-3)^{2} & \text { c) } 3(3 x+5)(3 x-5)\end{array}$
d) $(2 y+3)(3 y-5)$ e) $2 a(a+2)(a+5)$
f) not factorable; the parabola opens upward and has a minimum of 0.75 , so it never has a value of 0 ; there are no $x$-intercepts
4. a) vertex form; opens upward
b) $(2,-5)$; minimum, because the parabola opens upward
c) $x=2$
d) 7
e)

f) increasing for $x>2$; decreasing for $x<2$
5. a)

$\begin{array}{lll}\text { b) } 56 \mathrm{~m} & \text { c) } 49 \mathrm{~m} & \text { d) } 28 \mathrm{~m}\end{array}$
c)
