Name: _____ Class: _____ Date: _____ ID: A

MCV4U - Chapter 1 Review

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. For the function $f(x) = x^3 + 2x$, determine the average rate of change of $f(x)$ with respect to x over the interval $1 \le x \le 3$.

a. –30
b. –2
c. 2
d. 15

____ 2. For $f(x) = x^2 - 6$, determine the average rate of change of $f(x)$ with respect to x over the interval $-1 \le x \le 4$.

a. –3
b. 5
c. 3
d. 6

____ 3. For the function $y = f(x)$, what does the quotient $\frac{\Delta y}{\Delta x}$ represent?

a. the average rate of change for $f(x)$ over the interval $x_1 \le x \le x_2$
b. the derivative of f at x_1
c. the mean value of $f(x)$
d. the instantaneous rate of change for $f(x)$ at x_1

A function h has the following table of values.

x	–7.1	–7.01	–7.001	–7.0001	–7	–6.9999	–6.999	–6.99	–6.9
$h(x)$	54.2	54.1	54.001	54.0001	55	559 999.9	55 999.9	559.9	55.9

____ 4. Find the following: $\lim_{x \to -7^+} h(x)$

a. 55
b. ∞
c. 54
d. undefined

____ 5. Find the following: $\lim_{x \to -7} h(x)$

a. 55
b. ∞
c. 54
d. undefined

Knowledge

____ 6. The limit of the sequence $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{2^n}\right\}$ is

a. 1/2
b. infinity
c. 1
d. 0

____ 7. The limit of the sequence $\left\{\frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \ldots, \frac{(-1)^{n-1}}{n+1}, \ldots\right\}$ is

a. –1
b. a limit does not exist
c. 0
d. infinity

____ 8. Where is the function $f(x) = \tan x$ discontinuous?
a. Every multiple of $\pi / 2$
b. Every odd multiple of $\pi / 2$
c. Every even multiple of $\pi / 2$
d. The function is continuous everywhere.

Thinking

____ 9. The limit of the sequence $\left\{\sqrt{k}, \sqrt{k\sqrt{k}}, \sqrt{k\sqrt{k\sqrt{k}}}, \sqrt{k\sqrt{k\sqrt{k\sqrt{k}}}}, \ldots\right\}$ is

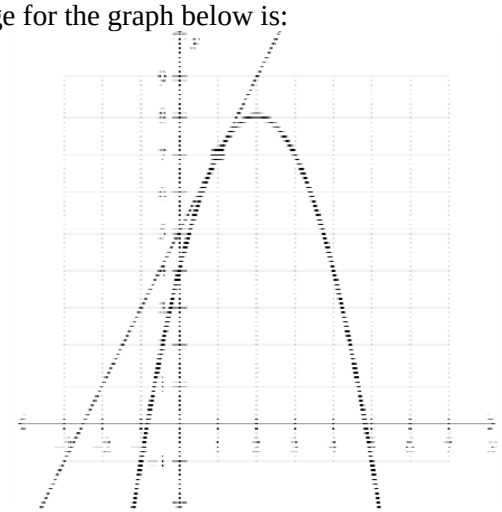
a. 1
b. k
c. 0
d. $-1/k$

____ 10. What can be said regarding the series $S_n = 1 + 1 + 1 + 1 + \ldots + 1$?
a. The series is arithmetic because a common difference exists.
b. The series is geometric, because a common ratio exists.
c. The sum of the terms do not approach a finite limit.
d. All of the above statements are true.

____ 11. For the function $y = f(x)$, what does the quotient $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ represent?

a. the mean value of $f(x)$
b. the slope of a secant line for $f(x)$
c. the derivative of f at x
d. the derivative of f at h

____ 12. The instantaneous rate of change for the graph below is:



a. 2
b. –2
c. $-2x + 5$
d. 3

____ 13. The limit of the sequence given by the expression: $\lim_{n \to \infty} \frac{3n^2}{2n^3 - 1}$, $n \in ..$ (natural numbers)

a. is 3/2
b. is 0
c. does not exist
d. is 3

____ 14. The limit of the sequence given by the expression: $\lim_{n \to \infty} \frac{\log n}{\sqrt{n+3}}$, $n \in ..$ (natural numbers)

a. is 1
b. is 0
c. does not exist
d. is negative

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____ 15. The limit of the sequence given by the expression: $\lim_{n \to \infty} \frac{n^3}{2n^2 + 3n + 5}$, $n \in ..$ (natural numbers)

a. is 1
b. is 2
c. does not exist
d. is negative

Short Answer

1. Determine whether the function f is continuous at $x = 3$:

$$f(x) = \begin{cases} \frac{x}{3} - 3, & x \geq 3 \\ (x-3)^2 - 2, & x < 3 \end{cases}$$

2. Determine whether the function f is continuous at $x = 5$:

$$f(x) = \begin{cases} \sqrt{x+4}, & x \leq 5 \\ -3x + 18, & x > 5 \end{cases}$$

3. Evaluate the limit, using limit laws: $\lim_{x \to 0} (2x^2 + 5x - 3)$

4. Evaluate the limit, using limit laws, showing any algebra: $\lim_{x \to \frac{\pi}{2}} \frac{\sin x + \cos x}{\tan x}$

5. Evaluate the limit, using limit laws, showing any algebra: $\lim_{x \to 1} \frac{x^2 - 1}{x + 1}$

6. Evaluate the limit, using limit laws, showing any algebra: $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x}$

7. A large balloon in the shape of a sphere has a volume $v(r) = \frac{4}{3}\pi r^3$. As the balloon is being inflated, find the average rate of change of the balloon's volume when $r = 1$, using first principles. (r is in metres).

8. An approximate model for the distance s metres traveled in t seconds by a skateboard going down a hill is $s = kt^2$, where k is a constant that depends on the slope of the hill. If, for one particular hill, the value of k is 0.1, find the instantaneous rate of change of distance with respect to time at 10 seconds using first principles.

9. State Cauchy's three conditions for continuous functions.

10. Show that $f(x)$ is discontinuous at $x = 0$ if $f(x) = \begin{cases} x - 1 & \text{if } x \leq -1 \\ x^2 & \text{if } -1 < x \leq 0 \\ (x+2)^2 & \text{if } x > 0 \end{cases}$ using Cauchy.

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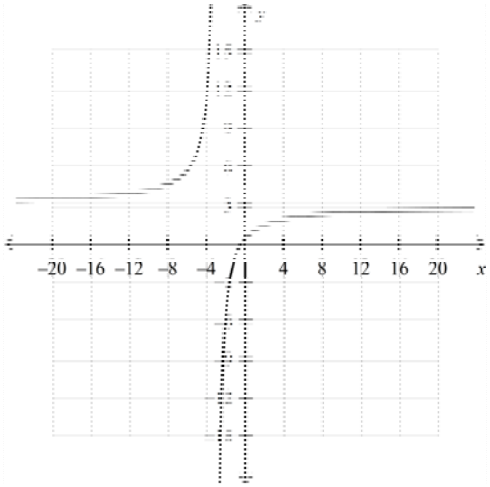
11. (Communication, 1) Given the graph of $f(x)$, determine $\lim_{x \to \infty} f(x)$.



12. What is the slope of the tangent of $f(x) = -3x^2$ at $x = \frac{1}{2}$?

Knowledge

State the rate of change of the given lines

13. $y = 3x - 2$

14. For $y = \frac{5}{2}x^2$, find the equation for the tangent slope for any x.

15. Find the instantaneous rate of change (exact) at $x = 2$ for the function $f(x) = x - 2x^3$.

16. Evaluate the limit using limit laws: $\lim_{x \to 2} \frac{3x}{x^2 + 2}$

Problem

1. From first principles, find the derivative of: $f(x) = x^{1/3} + x^{-1/3}$.

2. Find the derivative for $f(x) = \sqrt{9 + \sqrt{9 + x}}$ using first principles.

3. If a stone is thrown downward with a speed of 15 m/s from a cliff that is 80 m high, its height in metres after t seconds is $s(t) = 80 - 15t - 4.9t^2$.
   a) Determine when the stone hits the ground.
   b) Find the stone's instantaneous rate of change of distance when it hits the ground.

4. Show that the curve $y = x^2$ and the curve $y = \frac{1}{\sqrt{x}}$ intersect each other at right angles.

5. Evaluate the tangent of the function $f(x) = \frac{2}{\sqrt{x}}$ at the point where $x = a$.

MCV4U - Chapter 1 Review
Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 REF: Knowledge and Understanding
OBJ: 3.1 Examining Rates of Change in Polynomial Models STA: CC1.02
TOP: Rates of Change in Polynomial Function Models NOT: from Nelson MCB4U bank
2. ANS: C PTS: 1 REF: Knowledge and Understanding
OBJ: 3.1 Examining Rates of Change in Polynomial Models STA: CC1.02
TOP: Rates of Change in Polynomial Function Models NOT: from Nelson MCB4U bank
3. ANS: A PTS: 1 REF: Knowledge and Understanding
OBJ: 3.1 Examining Rates of Change in Polynomial Models STA: CC1.02
TOP: Rates of Change in Polynomial Function Models NOT: from Nelson MCB text
4. ANS: B PTS: 1 REF: Knowledge and Understanding
OBJ: 3.3 Limits of Polynomial Functions STA: DA1.01
TOP: Rates of Change in Polynomial Function Models
5. ANS: D PTS: 1 REF: Knowledge and Understanding
OBJ: 3.3 Limits of Polynomial Functions STA: DA1.01
TOP: Rates of Change in Polynomial Function Models
6. ANS: D PTS: 1 DIF: ** REF: K
STA: A.1.4 TOP: The Limit of a Sequence
NOT: PJK - From DDEK (Analysis 13, Gage), Sec 10.3, Page 343 #1b
7. ANS: C PTS: 1 DIF: ** REF: K
STA: A.1.4 TOP: The Limit of a Sequence
NOT: PJK - From DDEK (Analysis 13, Gage), Sec 10.3, Page 343 #1c
8. ANS: B PTS: 1 DIF: *** REF: K
STA: A.1.2 TOP: Continuity
NOT: PJK - From DDEK (Analysis 13, Gage), Sec 10.3, Page 362, a modified form of #2
9. ANS: B PTS: 1 DIF: ** REF: K
STA: A.1.4 TOP: The Limit of a Sequence
NOT: PJK - From DDEK (Analysis 13, Gage), Sec 10.3, Page 343, a modified form of #4
10. ANS: D PTS: 1 DIF: *** REF: T
STA: A.1.4 TOP: The Limit of a Sequence
NOT: PJK - From DDEK (Analysis 13, Gage), Sec 10.3, Page 343, a modified form of #5
11. ANS: C PTS: 1 REF: Knowledge and Understanding
OBJ: 3.5 Finding Some Shortcuts - The Constant and Power Rules
STA: DA2.01 TOP: Rates of Change in Polynomial Function Models
12. ANS: A PTS: 1 DIF: ** REF: K
TOP: Rate of Change NOT: PJK - based on text
13. ANS: B PTS: 1 DIF: ** TOP: 1.3 Limit of a sequence
NOT: PJK
14. ANS: B PTS: 1 DIF: ** TOP: 1.3 Limit of a sequence
NOT: PJK
15. ANS: C PTS: 1 DIF: ** TOP: 1.3 Limit of a sequence
NOT: PJK

SHORT ANSWER

1. ANS:
First, determine if the limit exists:

$$\lim_{x \to 3^+} f(x) = -2$$

$$\lim_{x \to 3^-} f(x) = -2$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$$

So, the limit is –2 at x = 3.
Second, determine the continuity.
The limit is –2
$f(3) = -2$
and the limit = $f(3)$.
Therefore, by Cauchy, the function is continuous.

PTS: 1 DIF: *** REF: T TOP: Continuity (Cauchy)
NOT: PJK - A piecewise continuity question.

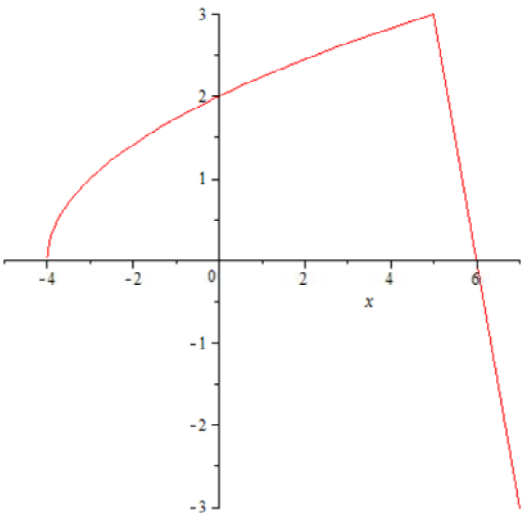
2. ANS:
A solution and graph are provided:
First, determine if the limit exists:

$$\lim_{x \to 5^+} f(x) = 3$$

$$\lim_{x \to 5^-} f(x) = 3$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^-} f(x)$$

So, the limit is 3 at x = 5.
Second, determine the continuity.
The limit is 3
$f(5) = 3$
and the limit = $f(5)$.
Therefore, by Cauchy, the function is continuous.



PTS: 1 DIF: *** REF: T TOP: Continuity (Cauchy)
NOT: PJK - A piecewise continuity question.

3. ANS:

$$\lim_{x \to 0} 2x^2 + 5 \lim_{x \to 0} x - \lim_{x \to 0} 3 = 2 \lim_{x \to 0} x^2 + 0 - 3$$

$$= 2\left(\lim_{x \to 0} x\right)^2 - 3$$

$$= -3$$

PTS: 1 DIF: ** REF: A TOP: Limit laws
NOT: PJK

4. ANS:

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin x+\cos x}{\tan x} = \lim_{x\to\frac{\pi}{2}} \frac{\cos x\sin x+\cos^2 x}{\sin x}$$

$$= \frac{\lim_{x\to\frac{\pi}{2}} \cos x \lim_{x\to\frac{\pi}{2}} \sin x + \lim_{x\to\frac{\pi}{2}} \cos x}{\lim_{x\to\frac{\pi}{2}} \sin x}$$

$$= \frac{0+0}{1} = 0$$

PTS: 1 DIF: *** REF: A TOP: Limit laws
NOT: PJK - Algebraic manipulation needed first to avoid undefined

5. ANS:

$$\lim_{x\to 1} \frac{x^2-1}{x+1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x+1}$$

$$= \lim_{x\to 1} (x-1)$$

$$= \lim_{x\to 1} x - \lim_{x\to 1} 1$$

$$= 0$$

PTS: 1 DIF: *** REF: A TOP: Limit laws
NOT: PJK - Algebraic manipulation needed first to avoid undefined. A beginning question from Spivak

6. ANS:

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x\to 1} \frac{1-\sqrt{x}}{\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}$$

$$= \lim_{x\to 1} \frac{1}{1+\sqrt{x}}$$

$$= \frac{\lim_{x\to 1} 1}{\lim_{x\to 1} 1+\sqrt{\lim_{x\to 1} x}}$$

$$= 1/2$$

PTS: 1 DIF: *** REF: A TOP: Limit laws
NOT: PJK - Algebraic manipulation needed first to avoid undefined. A beginning question from Spivak

7. ANS:

$$v'(r) = \lim_{h \to 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi\left(r^3 + 3r^2h + 3rh^2 + h^3 - r^3\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{3}\pi\left(3r^2h + 3rh^2 + h^3\right)}{h}$$

$$= \frac{4}{3}\pi \lim_{h \to 0}\left(3r^2 + 3rh + h^2\right)$$

$$= 4\pi r^2$$

So, that when $r = 1$, the volume increases at the rate of 4π m^3/(m radius) (about 12.568 m^3 per m radius).

PTS: 1 DIF: *** REF: Application OBJ: 1.5 Introduction to Derivatives
STA: A1.5 TOP: Introduction to Derivatives
NOT: PJK - a made-up question for change in volume

8. ANS:

$$s'(t) = \lim_{h \to 0} \frac{0.1(10+h)^2 - 0.1(10)^2}{h}$$

$$= \lim_{h \to 0} \frac{0.1(100 + 20h + h^2) - 0.1(10)^2}{h}$$

$$= \lim_{h \to 0} \frac{10 + 2h + h^2 - 10}{h}$$

$$= \lim_{h \to 0} 2 - h$$

$$= 2 \text{ m/s}$$

PTS: 1 REF: Application OBJ: 3.1 Examining Rates of Change in Polynomial Models
STA: CC1.01 | CC1.03 TOP: Rates of Change in Polynomial Function Models

9. ANS:

A function f is continuous at a if

1) $f(a)$ is defined
2) $\lim_{x \to a} f(x)$ exists
3) $\lim_{x \to a} f(x) = f(a)$

PTS: 1 DIF: ** REF: Knowledge and Understanding
STA: A.1.2 TOP: Definition of Continuity
NOT: PJK - Using the Cauchy definition of continuity, one that is rigorous enough to be used on almost all functions. From Analysis 13, by DDEK, Page 361.

10. ANS:
(1) $f(0) = 0^2 = 0$

(2) The left-hand limit does not include 0, so, we need to approximate:

$$\lim_{x \to 0^+} (x+2)^2 = (2.00001)^2$$

$$= 4.000004 \approx 4$$

$$\lim_{x \to 0^-} x^2 = (0)^2$$

$$= 0$$

$f(x)$ fails Cauchy's test for continuity at $x = 0$, since the limit does not exist (left and right-hand limits are not equal). Therefore, f is not continuous at $x = 0$.

PTS: 1 REF: Knowledge and Understanding OBJ: 1.4 Limits and Continuity
STA: A1.4 TOP: Limits and Continuity
NOT: PJK - Transmorgrified from NEL AFIC.

11. ANS:
3

PTS: 1 REF: Knowledge and Understanding
OBJ: 5.2 Limits and End Behaviour of Rational Functions STA: DA1.02
TOP: Rates of Change in Rational Function Models

12. ANS:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-3(x+h)^2 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{-6xh - 3h^2}{h}$$

$$= \lim_{h \to 0} (-6x - 3h)$$

$$= -6x$$

such that when $x = 1/2$, the tangent has slope -3

PTS: 1 REF: Knowledge and Understanding
OBJ: 3.4 Using Limits to Find Instantaneous Rates of Change - The Derivative
STA: DA1.04 TOP: Rates of Change in Polynomial Function Models
NOT: PJK -- Solution

13. ANS:

$$\lim_{h \to 0} \frac{3(x+h) - 2 - 3x + 2}{h} = \lim_{h \to 0} \frac{3x + 3h - 2 - 3x + 2}{h}$$

$$= \lim_{h \to 0} 3$$

$$= 3$$

PTS: 1 DIF: ** REF: K STA: A.1.4
TOP: Rate of Change
NOT: (PJK) -- DDEK (Analysis 13, Gage, 1966), similar to Q2 on page 374.

14. ANS:

$$\lim_{h\to 0}\frac{5(x+h)^2/2-5x^2/2}{h}=\lim_{h\to 0}\frac{5x^2/2+10xh/2+5h^2/2-5x^2/2}{h}$$

$$=\lim_{h\to 0}\frac{10xh/2+5h^2/2}{h}$$

$$=\lim_{h\to 0}(5x+5h/2)$$

$$=5x$$

PTS: 1 REF: Knowledge and Understanding
OBJ: 3.5 Finding Some Shortcuts - The Constant and Power Rules
STA: DA2.02 TOP: Rates of Change in Polynomial Function Models

15. ANS:

For this, we need the limit. It would be good to first expand $f(x+h)$:

$$f(x+h)=x+h-2(x+h)^3=x+h+2(x^3+3x^2h+3xh^2+h^3)$$

$$=x+h+2x^3+6x^2h+6xh^2+2h^3$$

Thus, $\lim_{h\to 0}\frac{x+h-2x^3-6x^2h-6xh^2-2h^3-x+2x^3}{h}=\lim_{h\to 0}\frac{h-6x^2h-6xh^2-2h^3}{h}$

$$=\lim_{h\to 0}1-6x^2-6xh-2h^2$$

$$=1-6x^2$$

PTS: 1 DIF: *** REF: A LOC: 1.5
TOP: 1.5 Introduction to derivatives NOT: PJK -- #11 p 20

16. ANS:

$$\lim_{x\to 2}\frac{3x}{x^2+2}=\frac{\lim_{x\to 2}3x}{\lim_{x\to 2}(x^2+2)}$$

$$=\frac{3\lim_{x\to 2}x}{\left(\lim_{x\to 2}x\right)^2+\lim_{x\to 2}2}$$

$$=\frac{3(2)}{2^2+2}$$

$$=1$$

PTS: 1 DIF: ** TOP: 1.3 Limits NOT: PJK

PROBLEM

1. ANS:
This is best done by breaking up the sum into two separate limits, and hence, derivatives.
A solution exists in the parametric equations, and relying on the factorzation of a difference of cubes. If your students were not expected to know the sum/difference of cubes formula, then I would suppose you would need to supply it, but it gives away the "secret", and it may be possible that students may still find it difficult.
Let $a = \sqrt[3]{x+h} = (x+h)^{1/3}$ and $b = x^{1/3}$.
For the negative explonents, let $c = (x+h)^{-1./3}$ and $d = x^{-1./3}$.

$$\frac{d}{dx}x^{1/3} + x^{-1/3} = \lim_{h\to 0}\frac{(x+h)^{1/3} - x^{1/3}}{h} + \lim_{h\to 0}\frac{(x+h)^{-1/3} - x^{-1/3}}{h}$$

$$= \lim_{h\to 0}\frac{a-b}{h} + \lim_{h\to 0}\frac{c-d}{h}$$

$$= \lim_{h\to 0}\frac{(a-b)}{h}\frac{(a^2+ab+b^2)}{(a^2+ab+b^2)} + \lim_{h\to 0}\frac{(c-d)}{h}\frac{(c^2+cd+d^2)}{(c^2+cd+d^2)}$$

$$= \lim_{h\to 0}\frac{a^3-b^3}{h(a^2+ab+b^2)} + \lim_{h\to 0}\frac{c^3-d^3}{h(c^2+cd+d^2)}$$

$$= \lim_{h\to 0}\frac{x+h-x}{h((x+h)^{2/3}+(x+h)^{1/3}x^{1/3}+x^{2/3})} + \lim_{h\to 0}\frac{(x+h)^{-1}-x^{-1}}{h(c^2+cd+d^2)}$$

$$= \frac{1}{4x^{2/3}} + \lim_{h\to 0}\frac{\dfrac{x}{x(x+h)} - \dfrac{x+h}{x(x+h)}}{h((x+h)^{-2/3}+(x+h)^{-1/3}x^{-1/3}+x^{-2/3})}$$

$$= \frac{1}{4x^{4/3}} + \lim_{h\to 0}\frac{-h}{h((x+h)^{-2/3}+(x+h)^{-1/3}x^{-1/3}+x^{-2/3})(x)(x+h)}$$

$$= \frac{1}{4x^{2/3}} + \lim_{h\to 0}\frac{-1}{((x+h)^{-2/3}+(x+h)^{-1/3}x^{-1/3}+x^{-2/3})(x)(x+h)}$$

$$= \frac{1}{4x^{4/3}} + \frac{-1}{(2x^{-2/3}+x^{-2/3})(x^2)}$$

$$= \frac{1}{4x^{2/3}} - \frac{1}{3x^{4/3}}$$

A conjugate exists and is $(x^{1/3} - x^{-1/3})$, and nearly looks promising, but repeated applications of this reveals that the exponent's numerator will always be a power of 2, and 3 will never divide any power of 2 evenly (you can prove this by induction).

PTS: 1 DIF: **** REF: T TOP: 1.5 Derivatives by First Principles
NOT: PJK - I couldn't solve this at first, and website research revealed that this had humbled many other teachers also. But easily solved by all teachers using the power rule :-)

2. ANS:

$$f'(x) = \frac{d}{dx}\sqrt{9+\sqrt{9+x}}$$

$$= \lim_{h\to 0}\frac{\sqrt{9+\sqrt{9+x+h}} - \sqrt{9+\sqrt{9+x}}}{h}$$

Let $a = \sqrt{9+x+h}$, and $b = \sqrt{9+x}$.

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{9+b} - \sqrt{9+a}}{h} \times \frac{\sqrt{9+b} + \sqrt{9+a}}{\sqrt{9+b} + \sqrt{9+a}}$$

$$= \lim_{h\to 0}\frac{9+b-9-a}{h(\sqrt{9+b} + \sqrt{9+a})}$$

$$= \lim_{h\to 0}\frac{b-a}{h(\sqrt{9+b} + \sqrt{9+a})}$$, substuting on the numerator:

$$= \lim_{h\to 0}\frac{\sqrt{9+x+h} - \sqrt{9+x}}{h(\sqrt{9+b} + \sqrt{9+a})} \times \frac{\sqrt{9+x+h} + \sqrt{9+x}}{b+a}$$

$$= \lim_{h\to 0}\frac{9+x+h-9-x}{h(\sqrt{9+b} + \sqrt{9+a})(b-a)}$$, cancelling h and substituting on the denominator:

$$= \lim_{h\to 0}\frac{1}{(\sqrt{9+\sqrt{9+x+h}} + \sqrt{9+\sqrt{9+x}})(\sqrt{9+x+h} + \sqrt{9+x})}$$

$$= \frac{1}{(\sqrt{9+\sqrt{9+x+0}} + \sqrt{9+\sqrt{9+x}})(\sqrt{9+x+0} + \sqrt{9+x})}$$

$$= \frac{1}{2\sqrt{9+\sqrt{9+x}}\,2\sqrt{9+x})} = \frac{1}{4\sqrt{9+\sqrt{9+x}}\,\sqrt{9+x})}$$

PTS: 1 DIF: **** REF: T TOP: 1.5 Derivatives by First Principles
NOT: PJK - Something observed as a problem from the 1979 Calculus text (Ontario 13). Verified in Maple.

3. ANS:

a) Use the graphing calculator to determine when $s(t) = 0$.

$t = 2.79$ s

b) Find $s'(t)$.

$$s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \to 0} \frac{\left(80 - 15(t+h) - 4.9(t+h)^2\right) - \left(80 - 15t - 4.9t^2\right)}{h}$$

$$= \lim_{h \to 0} \frac{80 - 15t - 15h - 4.9(t^2 + 2th + h^2) - 80 + 15t + 4.9t^2}{h}$$

$$= \lim_{h \to 0} \frac{-15h - 4.9t^2 - 9.8th - 4.9h^2 + 4.9t^2}{h}$$

$$= \lim_{h \to 0} \frac{-15h - 9.8th - 4.9h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-15 - 9.8t - 4.9h)}{h}$$

$$= \lim_{h \to 0} (-15 - 9.8t - 4.9h)$$

$s'(t) = -15 - 9.8t = -15 - 9.8(2.79) = -42.342$

The instantaneous rate of change is down at a rate of –42.34 m/s.

PTS: 1 REF: Application
OBJ: 3.4 Using Limits to Find Instantaneous Rates of Change - The Derivative
STA: CC2.04 | DA1.04 TOP: Rates of Change in Polynomial Function Models

4. ANS:
Find the point of intersection.

$$y = x^2 = \frac{1}{\sqrt{x}}$$

$x = 1$
$y = 1$

Find the slope of the tangents to the curves at the point (1, 1).

$y' = 2x$

At $x = 1$, $y' = 2$

$$y' = -\frac{1}{2}x^{-\frac{3}{2}}$$

At $x = 1$, $y' = -\frac{1}{2}$

Using first principles on $y = x^2$, letting $x = 1$:

$$f'(1) = \lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} 2 + h$$

$$= 2$$

The slope at $x = 1$ is 2.
Both tangent slopes at the point where the two functions meet are negative reciprocals of each other. Therefore, both tangent lines intersect at right angles (are perpendicular to each other).

Using first principles: (on $y = 1/\sqrt{x}$ when $x = 1$)

$$f'(1) = \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+h}} - \frac{1}{\sqrt{1}}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{1+h}}{1+h} - 1\right)\frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h}}{h(1+h)} - \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h} - (1+h)}{h(1+h)} = \lim_{h \to 0} \frac{\sqrt{1+h} - \left(\sqrt{1+h}\right)^2}{h\left(\sqrt{1+h}\right)^2}$$

$$= \lim_{h \to 0} \left(\frac{1 - \sqrt{1+h}}{h\sqrt{1+h}}\right)\left(\frac{1 + \sqrt{1+h}}{1 + \sqrt{1+h}}\right)$$

$$= \lim_{h \to 0} \frac{1 - 1 - h}{h\sqrt{1+h}\left(1 + \sqrt{1+h}\right)}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{1+h}\left(1 + \sqrt{1+h}\right)}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{1+h}\left(1 + \sqrt{1+h}\right)} = \frac{-1}{1(1+1)} = \frac{-1}{2}$$

The slope at $x = 1$ is –1/2.

PTS: 1 DIF: *** REF: Thinking/Inquiry/PS
OBJ: 3.6 Finding Some Shortcuts - The Sum and Difference Rules
STA: DA4.01
TOP: Derivatives by First Principles | Rates of Change in Polynomial Function Models
NOT: PJK -- First principles answers are mine. Students found this a challenging question if first principles were used.

5. ANS:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2(x+h)^{-1/2}-2x^{-1/2}}{h}\times\frac{2(x+h)^{-1/2}+2x^{-1/2}}{2(x+h)^{-1/2}+2x^{-1/2}}$$

$$= \lim_{h\to 0}\frac{\dfrac{4}{x+h}-\dfrac{4}{x}}{h\left(\dfrac{2}{\sqrt{x+h}}+\dfrac{2}{\sqrt{x}}\right)} = \lim_{h\to 0}\frac{\dfrac{4x-4x-4h}{x(x-h)}}{h\left(\dfrac{2}{\sqrt{x+h}}+\dfrac{2}{\sqrt{x}}\right)}$$

$$= \lim_{h\to 0}\frac{\dfrac{-4}{x(x-h)}}{\dfrac{2}{\sqrt{x+h}}+\dfrac{2}{\sqrt{x}}} = \frac{\dfrac{-4}{x(x-0)}}{\dfrac{2}{\sqrt{x+0}}+\dfrac{2}{\sqrt{x}}}$$

$$= \frac{-4/x^2}{4/\sqrt{x}}$$

$$= -1/x^{3/2}$$

by substitution, the tangent of the function is $-a^{\frac{-3}{2}}$

PTS: 1 DIF: **** REF: Knowledge and Understanding
OBJ: 3.5 Finding Some Shortcuts - The Constant and Power Rules
STA: DA2.02 TOP: Rates of Change in Polynomial Function Models
NOT: PJK -- a square root problem