Chapter 8 Prerequisite Skills

Linear Relations
1. Make a table of values and graph each linear function.
   a) \( y = 2x - 3 \) 
   b) \( y = -3x + 5 \) 
   c) \( 2x + 6y = 12 \) 
   d) \( 3x + 7y = 21 \)

2. Find the x- and y-intercepts of each linear function.
   a) \( y = -2x + 5 \)
   b) \( y = 5x + 20 \)
   c) \( 2x - 4y = 12 \)
   d) \( 3x + 6y = 18 \)

3. Graph each line.
   a) slope = -3; y-intercept = 2 
   b) slope = 2; y-intercept = -5 
   c) slope = \(-\frac{1}{2}\), through (-5, 8) 
   d) slope = 4, through (-2, -5) 
   e) \( x - 3 = 0 \) 
   f) \( 4y + 16 = 0 \)

Solving Linear Systems
4. Determine the coordinates of the point of intersection of each linear system.
   a)
   ![Graph](image1)
   b)
   ![Graph](image2)

5. Solve each linear system.
   a) \( 3x + 2y = 9 \quad x - 2y = 7 \)
   b) \( 8x + 5y = 9 \quad 6x + 9y = 5 \)
   c) \( x + 3y = 5 \quad -x - 7y = 3 \)
   d) \( 9x + 5y = 53 \quad 5x - 6y = -32 \)

Writing Equations of Lines
6. Write the equation of each line.
   a) parallel to \( 3x - y + 7 = 0 \) with y-intercept 4 
   b) parallel to \( 2x + 3y = 9 \) and through P(-1, 6) 
   c) perpendicular to \( 3x + 5y = 8 \) with the same y-intercept as \( 5x - 3y = 12 \) 
   d) perpendicular to \( 2x + 3y = 5 \) and through Q(2, -7) 

Dot and Cross Products
7. Use \( \vec{a} \cdot \vec{b} \) to determine if \( \vec{a} \) and \( \vec{b} \) are perpendicular.
   a) \( \vec{a} = [2, -1] \), \( \vec{b} = [-1, 3] \) 
   b) \( \vec{a} = [1, 2] \), \( \vec{b} = [4, 5] \) 
   c) \( \vec{a} = [1, 1] \), \( \vec{b} = [-1, 1] \) 
   d) \( \vec{a} = [2, 3, 1] \), \( \vec{b} = [1, 2, -8] \) 
   e) \( \vec{a} = [1, 2, 3] \), \( \vec{b} = [3, -4, 2] \) 
   f) \( \vec{a} = [-1, 3, 4] \), \( \vec{b} = [1, 3, 5] \)

8. Find \( \vec{a} \times \vec{b} \).
   a) \( \vec{a} = [3, -2, 7] \), \( \vec{b} = [-1, 4, -5] \) 
   b) \( \vec{a} = [-5, 6, -7] \), \( \vec{b} = [2, -7, 4] \)

9. Find a parallel vector and perpendicular vector to each given vector.
   a) \( \vec{a} = [1, -2] \) \quad b) \( \vec{b} = [5, 7] \)
   c) \( \vec{c} = [3, -2, 5] \) \quad d) \( \vec{d} = [-2, 7, -1] \) 

10. Find the measure of the angle between the vectors in each pair.
    a) \( \vec{a} = [2, 5] \), \( \vec{b} = [3, -1] \) 
    b) \( \vec{a} = [9, 12] \), \( \vec{b} = [-11, 15] \) 
    c) \( \vec{a} = [1, -2, 3] \), \( \vec{b} = [4, -3, 2] \) 
    d) \( \vec{a} = [5, -3, 2] \), \( \vec{b} = [1, 0, 4] \)
8.1 Equations of Lines in Two-Space and Three-Space

1. Write a vector equation for a line given each direction vector \( \vec{m} \) and point \( P_0 \).
   a) \( \vec{m} = [2, -7], P_0(9, 4) \)
   b) \( \vec{m} = [-1, 5], P_0(3, 7) \)
   c) \( \vec{m} = [2, -3, 5], P_0(8, -11, 2) \)
   d) \( \vec{m} = [-3, 4, 7], P_0(7, -1, 4) \)

2. Write a vector equation of the line that passes through each pair of points.
   a) \( A(2, 3), B(7, 1) \)
   b) \( A(5, -1), B(-2, 7) \)
   c) \( A(4, -3, 1), B(2, -3, 7) \)
   d) \( A(5, 6, -1), B(0, -3, 2) \)

3. Determine if each point \( P \) is on the line \( [x, y] = [3, 1] + t[2, 5] \).
   a) \( P(12, 36) \)
   b) \( P(-7, 26) \)
   c) \( P(1, 6) \)
   d) \( P(-3, 15) \)

4. Write the parametric equations for each vector equation.
   a) \( [x, y] = [7, 3] + t[2, 5] \)
   b) \( [x, y] = [-1, 4] + t[8, -9] \)
   c) \( [x, y, z] = [0, 4, -7] + t[3, -4, 1] \)
   d) \( [x, y, z] = [7, 5, -4] + t[-2, 1, 3] \)

5. Write a vector equation for each line, given the parametric equations.
   a) \( x = 2 - 4t \)
      \( y = 1 - 5t \)
   b) \( x = 8 - 9t \)
      \( y = 3 \)
   c) \( x = 4 + 3t \)
      \( y = 7 - 2t \)
      \( z = 1 + t \)
   d) \( x = 4t \)
      \( y = 5 - 9t \)
      \( z = -3 \)

6. Given each set of parametric equations, write the scalar equation.
   a) \( x = 3 + 4t \)
      \( y = -1 - 5t \)
   b) \( x = 2 + 8t \)
      \( y = -5 + 7t \)

7. Write the scalar equation of each line given the normal vector \( \vec{n} \) and point \( P_0 \).
   a) \( \vec{n} = [4, 1], P_0(3, -5) \)
   b) \( \vec{n} = [1, -3], P_0(4, 3) \)
   c) \( \vec{n} = [-2, 6], P_0(2, -1) \)
   d) \( \vec{n} = [7, 0, 1], P_0(-1, 5, -2) \)
   e) \( \vec{n} = [-3, 4, 1], P_0(-7, -2, 0) \)

8. Write a vector equation and the \( \vec{m} \) parametric equations of a line going through the points \( A(7, 8, -3) \) and \( B(-2, 3, 5) \).

9. Determine which points are on the line \( [x, y, z] = [3, 1, -4] + t[2, 0, 5] \).
   a) \( (-5, 1, -24) \)
   b) \( (19, 1, 37) \)
   c) \( (9, 1, 11) \)
   d) \( (-17, -1, -33) \)

10. Determine the vector equation of each line.
    a) \( \) parallel to the \( y \)-axis and through \( P_0(2, -7) \)
    b) \( \) parallel to \( [x, y] = [3, -1] + t[3, 4] \) and through \( P_0(-2, 4) \)
    c) \( \) perpendicular to \( [x, y] = [2, -1] + t[5, 3] \) with \( x \)-intercept 3
    d) \( \) through \( P_0(-1, -7, 7) \) and perpendicular to \( [x, y, z] = [1, -7, 3] + t[4, -1, 2] \)
8.2 Equations of Planes

1. Does each point lie on the plane
   \[ 3x - 5y - 6z = 12 \]?
   a) A(0, 0, -2)
   b) B(1, 1, -2)
   c) C(1, -3, 1)
   d) D(-6, 0, 1)

2. Find the x-, y-, and z-intercepts of each plane.
   a) \[ 2x - 3y + 6z = 12 \]
   b) \[ x + 7y + 8z = 56 \]
   c) \[ 5x - 3y + 15z = 15 \]
   d) \[ 4x - 8z = 16 \]

3. Write the parametric equations of each plane given its vector equation.
   a) \[ [x, y, z] = [3, -1, 2] + t[4, -2, 3] + s[5, 1, 0] \]
   b) \[ [x, y, z] = [0, 8, 7] + t[1, 0, -3] + s[1, -4, 7] \]
   c) \[ [x, y, z] = [3, 6, -4] + t[4, -8, 5] + s[8, -9, 5] \]

4. Write the vector equation of a plane given its parametric equations.
   a) \[ x = 1 - 9t + 4s \]
      \[ y = 8 - 7t + s \]
      \[ z = -1 - 3t + 2s \]
   b) \[ x = 4 - t - s \]
      \[ y = 3 + 4s \]
      \[ z = 2t \]
   c) \[ x = 4 + t \]
      \[ y = 3 - s \]
      \[ z = 1 \]

5. Determine if each point is on the plane
   \[ [x, y, z] = [-3, 2, 4] + t[1, -3, 5] + s[-2, 1, 0] \].
   a) P(5, -7, 13)
   b) P(-6, 1, 9)
   c) P(-5, 4, -6)
   d) P(-10, 9, -10)

6. Determine the x-, y-, and z-intercepts of each plane.
   a) \[ [x, y, z] = [1, -3, 2] + t[4, -3, 5] + s[-1, 7, 0] \]
   b) \[ [x, y, z] = [9, -7, 4] + t[0, 3, -1] + s[5, -1, 1] \]

7. Write a vector equation for each plane.
   a) contains the origin; has direction vectors \( \vec{a} = [2, -1, 7] \) and \( \vec{b} = [3, 5, 2] \)
   b) contains the points D(1, -2, 3), E(5, -1, 8), and F(3, 9, 2)
   c) contains the point \( P_0(2, -1, 5) \); parallel to the \( xy \)-plane
   d) has y-intercept -7; parallel to the plane defined by the parametric equations
      \[ x = 7 + 3t \]
      \[ y = 6 + 2t - 5s \]
      \[ z = 1 - 8t + 3s \]

8. Write a scalar equation of the line that goes through the point (5, 2, 4) and is perpendicular to both
   \[ [x, y, z] = [-8, 5, 7] + t[-2, 8, 7] \] and
   \[ [x, y, z] = [1, -3, 2] + t[9, -1, 3] \].

9. Determine the vector equation of the plane that contains the points A(2, -1, 4), B(-3, 4, 5), and C(8, -1, 6).
8.3 Properties of Planes

1. Determine if each point lies on the plane
   \( 2x - 3y + 7z - 1 = 0 \).
   a) \((-1, -1, 0)\)  
   b) \((-1, 1, 1)\)  
   c) \((0, 2, 1)\)  
   d) \((2, -1, -1)\)

2. Write the scalar equation of each plane given the normal \( \mathbf{n} \) and a point \( P \) on the plane.
   a) \( \mathbf{n} = [1, -2, 3] \)  
      \( P (3, 1, 0) \)
   b) \( \mathbf{n} = [7, 8, -9] \)  
      \( P (-1, 1, 1) \)
   c) \( \mathbf{n} = [3, -7, 2] \)  
      \( P (-2, 5, -3) \)
   d) \( \mathbf{n} = [-2, 1, 5] \)  
      \( P (1, 2, 3) \)
   e) \( \mathbf{n} = [1, -3, -4] \)  
      \( P (-2, 5, 7) \)
   f) \( \mathbf{n} = [0, -3, 5] \)  
      \( P (8, -7, 3) \)

3. Find two vectors normal to each plane.
   a) \( 4x - 7y + 2z - 5 = 0 \)
   b) \(-9x + 5y - 4z - 1 = 0 \)
   c) \( 7y + 6x + 3 = 0 \)
   d) \( 3x - 8z = 0 \)
   e) \( x + 4y = 6z - 11 \)
   f) \( x = 4 \)

4. Write a scalar equation of each plane, given its vector equation.
   a) \( \mathbf{x} = \mathbf{a} + t\mathbf{d} \)
      \( \mathbf{a} = [7, -3, 4] \) and \( \mathbf{d} = [-1, -2, -3] \)
   b) \( \mathbf{x} = \mathbf{a} + t\mathbf{d} \)
      \( \mathbf{a} = [2, -3, 5] \) and \( \mathbf{d} = [-3, 4, 7] \)
   c) \( \mathbf{x} = \mathbf{a} + t\mathbf{d} \)
      \( \mathbf{a} = [1, 0, 3] \) and \( \mathbf{d} = [4, -6, 1] \)
   d) \( \mathbf{x} = \mathbf{a} + t\mathbf{d} \)
      \( \mathbf{a} = [7, -5, 11] \) and \( \mathbf{d} = [-1, 0, 7] \)

5. Write a scalar equation of each plane, given its parametric equations.
   a) \( \pi_1 \)
      \( x = 4 + t + 2s \)
      \( y = 3 - 2t - 3s \)
      \( z = -1 + 3t + s \)
      \( \pi_2 \)
      \( x = 5 - 3t + 2s \)
      \( y = 7 + 5t - s \)
      \( z = 2 - 4t + 5s \)

6. For each situation, write a scalar equation of the plane.
   a) has normal \( \mathbf{n} = (7, 9, -1) \) and includes the point \((3, -2, 4)\)
   b) contains direction vectors \( \mathbf{a} = (-1, 2, 8) \) and \( \mathbf{b} = (2, -1, 3) \) and includes the point \((2, -7, 8)\)
   c) parallel to the \( xy \)-plane and includes the point \((7, 8, -1)\)
   d) contains the points \((3, 8, -1), (-8, 9, -4), \) and \((1, -3, 2)\)
   e) contains the line \( \mathbf{x} = [4, -3, -2] + s[3, -2, 1] \) and parallel to the line defined by the parametric equations \( x = 5 + 3s \) and \( y = 1 - s \) and \( z = 2 + 4s \)
   f) contains the point \((2, -1, 8)\) and perpendicular to the line \( \mathbf{x} = [1, -2, -3] + s[5, -4, 7] \)
   g) parallel to the plane \(-3x + 2y + 5z + 8 = 0\) and includes the point \((5, -7, 8)\)
   h) contains the lines \( \mathbf{x} = [4, -1, 0] + s[-2, 1, 3] \) and \( \mathbf{x} = [-2, 4, 3] + s[-6, 5, 7] \)
8.4 Intersections of Lines in Two-Space and Three-Space

1. Solve each linear system in two-space.
   a) \(3x - 5y = -9\)
   \(4x + 5y = 23\)
   b) \(x - 2y = 7\)
   \(3x + 4y = 1\)
   c) \([x, y] = [5, 4] + s[-3, 1]\)
   \([x, y] = [2, 2] + t[2, -1]\)
   d) \([x, y] = [2, 6] + s[2, -3]\)
   \([x, y] = [6, 5] + t[1, 1]\)
   e) \([x, y] = [0, 2] + s[2, 3]\)
   \([x, y] = [7, -4] + t[1, 4]\)
   f) \([x, y] = [2, 3] + s[5, -4]\)
   \([x, y] = [-9, 5] + t[3, 1]\)
   g) \([x, y] = [1, 1] + s[-2, -1]\)
   \([x, y] = [5, -8] + t[4, -3]\)
   h) \([x, y] = [0, 7] + s[3, 0]\)
   \([x, y] = [-2, 3] + t[2, 1]\)

2. Determine if the parallel lines in each pair are distinct or coincident.
   a) \([x, y, z] = [5, -2, -8] + s[-3, 2, 5]\)
   \([x, y, z] = [-4, 0, 2] + t[-3, 2, 5]\)
   b) \([x, y, z] = [16, -8, 4] + s[4, -2, 1]\)
   \([x, y, z] = [4, -2, 1] + t[-4, 2, -1]\)
   c) \([x, y, z] = [-3, 0, -6] + s[-3, 0, -6]\)
   \([x, y, z] = [9, 1, 18] + t[9, 0, 18]\)
   d) \([x, y, z] = [10, -20, -15] + s[4, -8, -6]\)
   \([x, y, z] = [-18, 36, 27] + t[6, -12, -9]\)

3. Triangle ABC is formed from the intersections of the three lines represented by these equations.
   \(\ell_1 : [x, y] = [1, -3] + t[0, 1]\)
   \(\ell_2 : [x, y] = [2, 4] + s[-1, 6]\)
   \(\ell_3 : [x, y] = [2, 3] + r[1, 7]\)
   Find the length of each side of \(\Delta ABC\).

4. Parallelogram ABCD has vertices \(A(-1, -4), B(1, -3), C(6, -6), \) and \(D(4, -7)\). Find the vector equations of its diagonals and the point of intersection of the diagonals.

5. Determine if the lines in each pair intersect.
   If so, find the co-ordinates of the point of intersection.
   a) \([x, y] = [9, -1, 1] + s[-3, 4, 1]\)
   \([x, y] = [-3, 11, 5] + t[-3, 4, 1]\)
   b) \([x, y] = [1, 4, 5] + s[3, 0, -2]\)
   \([x, y] = [9, 4, -3] + s[3, 0, -2]\)
   c) \([x, y] = [1, 0, -3] + t[3, 5, 4]\)
   \([x, y] = [0, -9, -1] + s[-1, 2, -3]\)
   d) \([x, y] = [6, -4, 3] + t[-2, -1, 1]\)
   \([x, y] = [4, -1, 2] + s[2, 1, -1]\)
   e) \([x, y] = [-2, 0, -3] + t[5, 1, 3]\)
   \([x, y] = [5, 8, -6] + s[-1, 2, -3]\)

6. Determine the distance between the skew lines in each pair.
   a) \(\ell_1 : [x, y, z] = [4, -3, 2] + s[2, 7, -1]\)
   \(\ell_2 : [x, y, z] = [-2, 5, 4] + t[-4, 0, 3]\)
   b) \(\ell_1 : [x, y, z] = [-1, 6, 1] + s[-2, 4, 3]\)
   \(\ell_2 : [x, y, z] = [5, 1, 9] + t[3, -2, 4]\)
   c) \(\ell_1 : [x, y, z] = [6, 2, -1] + s[5, 3, -5]\)
   \(\ell_2 : [x, y, z] = [0, 4, 2] + t[2, -1, 1]\)
   d) \(\ell_1 : [x, y, z] = [-4, -1, -2] + s[-3, 0, 2]\)
   \(\ell_2 : [x, y, z] = [-1, -3, 0] + t[0, -5, -3]\)
8.5 Intersections of Lines and Planes

1. In each case, determine if the line and the plane are parallel.
a) \[ \ell_1 : \begin{align*} x &= 4 + 2t \\ y &= -t \\ z &= -1 - 4t \end{align*} \]
\[ \pi_1 : 3x + 2y + z - 7 = 0 \]
b) \[ \ell_2 : \begin{align*} x &= t \\ y &= 2t \\ z &= 3t \end{align*} \]
\[ \pi_2 : x - y + 2z = 5 \]

2. In each case, determine if the plane and line intersect. If so, state the solution.
a) \[ [x, y, z] = [1, 2, 5] + t[1, -1, 2] \]
\[ 2x + 6y - z = 5 \]
b) \[ [x, y, z] = [6, 11, 1] + t[1, 5, 2] \]
\[ x + 3y + 2z - 1 = 0 \]
c) \[ [x, y, z] = [9, 8, 3] + t[2, 1, 5] \]
\[ z = 0 \]
d) \[ [x, y, z] = [4, 2, 6] + t[1, -2, 3] \]
\[ 2x + 5y - z - 34 = 0 \]
e) \[ [x, y, z] = [3, 2, -1] + t[-2, 1, 3] \]
\[ x + 2y - 3z = 10 \]
f) \[ [x, y, z] = [4, 2, 6] + t[1, -2, 3] \]
\[ -4x - 5y + 6z = 34 \]

3. Find the distance between the parallel line and the plane.
a) \( \ell : [x, y, z] = [-5, 0, 1] + t[-2, 4, 7] \)
\[ \pi : 5x - 8y + 6z = 0 \]
b) \( \ell : [x, y, z] = [6, 2, -3] + t[3, 0, 1] \)
\[ \pi : 2x + 3y - 6z + 4 = 0 \]
c) \( \ell : [x, y, z] = [1, 3, 0] + t[-4, -5, 3] \)
\[ \pi : 2x - y + z = 6 \]
d) \( \ell : [x, y, z] = [0, 1, 1] + t[-1, 1, 0] \)
\[ \pi : -x - y + 12z = 24 \]

4. Find the distance between the planes.
a) \( \pi_1 : 2x + 2y - z - 3 = 0 \)
\( \pi_2 : 4x + 4y - 2z + 9 = 0 \)
b) \( \pi_1 : 2x - 4y + 2z - 1 = 0 \)
\( \pi_2 : 2x - 4y + 2z - 3 = 0 \)
c) \( \pi_1 : x + 2y + z = 4 \)
\( \pi_2 : x + 2y + z = -8 \)
d) \( \pi_1 : 4x - 12y + 6z + 7 = 0 \)
\( \pi_2 : 2x - 6y + 3z - 6 = 0 \)

5. Find the distance between each point and the given plane.
a) \( P(1, 1, -1) \)
\[ x + y - z - 3 = 0 \]
b) \( P(1, 2, 3) \)
\[ 2x + y - 2z - 4 = 0 \]
c) \( P(7, -3, 2) \)
\[ 2x - 3z - 1 = 0 \]
d) \( P(0, 0, 0) \)
\[ y = 7 \]
e) \( P(1, 2, -3) \)
\[ 2x - 3y - 6z + 14 = 0 \]
f) \( P(1, 3, -2) \)
\[ 4x - y - z + 6 = 0 \]
g) \( P(0, 0, 0) \)
\[ 5x + 3y - 2z - 37 = 0 \]
h) \( P(3, -1, 4) \)
\[ 6x - z - 11 = 0 \]

6. Determine the distance from point \( P(-2, -1, 1) \) to the plane \( [x, y, z] = [4, -1, 6] + t[1, 6, 3] + s[-2, 3, 1] \).
8.6 Intersection of Planes

1. If possible, determine the line through which the planes in each pair intersect.
   a) \(3x - 2y + z = 4\)
      \(6x - 4y + 3z = 7\)
   b) \(2x - 8y - 6z = 2 = 0\)
      \(-x + 4y + 3z = 0\)
   c) \(y = 4x - 2z + 3\)
      \[x = \frac{1}{4}y + \frac{1}{2}z\]

2. For each system of equations, determine the point of intersection.
   a) \(x + y + 2z = 5\)
      \(4x - 3y + z = -8\)
      \(-5x - 2y + 3z = 7\)
   b) \(x + y + 2z = 5\)
      \(4x - 3y + z = -78\)
      \(-5x - 2y + 3z = 27\)
   c) \(x + y - 3z = -1\)
      \(x - y = 3\)
      \(y + 2z = 5\)
   d) \(x + y + 2z = 5\)
      \(4x - 3y + z = 57\)
      \(-5x - 2y + 3z = -8\)
   e) \(2x + y - z = 1\)
      \(x + 3y + z = 10\)
      \(x + 2y - 2z = -1\)
   f) \(x + 4y + 3z = 5\)
      \(x + 3y + 2z + 4 = 0\)
      \(x + y - z = -1\)

3. Determine the line of intersection of each system of equations.
   a) \(x + 2y + 3z = -4\)
      \(x - y - 3z = 8\)
      \(x + 5y + 9z = -16\)
   b) \(x + 2y + 3z = 4\)
      \(2x + 3y + 4z = -5\)
      \(3x + 4y + 5z = -6\)
   c) \(x + y + z = -3\)
      \(2x + 2y - 3z = 4\)
      \(3x + 3y - 2z = 1\)
   d) \(3x - 2y + 5z = 1\)
      \(5x + y - 3z = -4\)
      \(x - 18y + 47z = 23\)
   e) \(3x - 2y + 5z = 1\)
      \(5x + y - 3z = -4\)
      \(x - 5y + 13z = 6\)

4. Determine if each system of planes is consistent or inconsistent. If possible, solve the system.
   a) \(2x + y + z = 6\)
      \(5x - y + 3z = 10\)
      \(x - 3y + z = -2\)
   b) \(4x + 4y - z = 8\)
      \(2x + 2y + z = 5\)
   c) \(x + y - z = 1\)
      \(x + 3y + z = 2\)
      \(x + 5y + 3z = 3\)
   d) \(11x + 10y + 9z = 5\)
      \(x + 2y + 3z = 1\)
      \(3x + 2y + z = 1\)
   e) \(x - 4y - 13z = 4\)
      \(x - 2y - 3z = 2\)
      \(-3x + 5y + 4z = 2\)
   f) \(4x + y + 3z = 7\)
      \(x - y + 2z = 3\)
      \(3x + 2y + z = 4\)

5. Describe each system of planes. If possible, solve the system.
   a) \(-x + y + 3z = 2\)
      \(2x - 2y - 6z = -4\)
      \(-3x + 3y + 9z = 6\)
   b) \(x = 0\)
      \(y = 0\)
      \(x + y = 4\)
   c) \(-x + y + 3z = 2\)
      \(x - 3y + 5z = 6\)
      \(-2y + 8z = 8\)
   d) \(-x + y + 3z = 2\)
      \(-x + y + 3z = 4\)
      \(2x - 2y - 6z = 10\)
   e) \(-x + y + 3z = 2\)
      \(-x + y + 3z = 4\)
      \(x - 3y + 5z = 6\)
   f) \(x = 0\)
      \(y = 0\)
      \(z = 0\)
   g) \(-x + y + 3z = 2\)
      \(2x - 2y - 6z = -4\)
      \(x - 3y + 5z = 6\)
Chapter 8 Review

8.1 Equations of Lines in Two-Space and Three-Space
1. Write the vector and parametric equations of each line.
   a) \( \vec{m} = [3, 5] \), \( P(4, -5) \)
   b) \( \vec{m} = [3, 7, -2] \), \( P(-6, 2, 1) \)
   c) perpendicular to the \( yz \)-plane and through \( (0, 1, 2) \)
   d) through the points \( A(4, -5, 3) \) and \( B(3, -7, 1) \)

2. Write the scalar equation for each line.
   a) \( [x, y] = [4, -3] + t[-1, 5] \)
   b) \( [x, y] = [2, -5] + t[2, -3] \)

3. A line is defined by the equation \( [x, y, z] = [-2, 3, 7] + t[3, -2, 5] \). Write the parametric equations for the line and determine if it contains the point \((10, -5, 22)\).

8.2 Equations of Planes
4. Find three points on each plane.
   a) \( [x, y, z] = [2, 4, -8] + t[1, 4, 2] + s[4, -5, 2] \)
   b) \( x = 3t - 4s \)
      \( y = 1 - t \)
      \( z = 5 + 2t - 4s \)
   c) \( 3x - 4y + z + 12 = 0 \)

5. Write the vector and parametric equations of each plane.
   a) contains the points \( D(1, 7, 2) \), \( E(4, 0, -1) \), and \( F(1, 2, 3) \).
   b) parallel to the \( xz \)-plane and through the point \( Q(2, -3, 4) \).

8.3 Properties of Planes
6. Write the scalar equation of the plane with \( \vec{n} = [2, -4, 3] \) that contains the point \( R(3, -5, 1) \).

7. Write the scalar equation of this plane \( [x, y, z] = [2, 1, 4] + t[-2, 5, 3] + s[1, 0, -5] \)

8. Write the scalar equation of each plane.
   a) contains the points \( A(1, 2, 3) \), \( B(2, 3, 4) \), and \( C(4, 5, 5) \)
   b) perpendicular to the \( xz \)-plane with \( z \)-intercept \(-1\)

8.4 Intersections of Lines in Two-Space and Three-Space
9. Determine the number of solutions of each linear system in two-space. If possible, solve each system.
   a) \( [x, y] = [-1, -4] + t[1, -1] \)
      \( x = 3 - 2t \)
      \( y = -1 + 3t \)
   b) \( y = \frac{2x - 1}{3} \)
      \( 2x + 3y + 1 = 0 \)

10. Determine if the lines in each pair intersect. If so, find the coordinates of the point of intersection.
    a) \( [x, y, z] = [3, -2, 3] + t[-1, 1, 2] \)
       \( [x, y, z] = [1, -1, 4] + s[1, 1, 4] \)
    b) \( [x, y, z] = [3, -3, 0] + t[3, -1, 1] \)
       \( [x, y, z] = [4, 0, 4] + s[-1, 1, 1] \)

11. Find the distance between these two skew lines.
    \( [x, y, z] = [2, 5, 3] + t[2, 1, -1] \)
    \( [x, y, z] = [3, 3, 1] + s[0, 2, 1] \)

8.5 Intersections of Lines and Planes
12. Determine if each line intersects the plane. If so, state the solution.
    a) \( [x, y, z] = [2, 5, 3] + t[1, 4, -2] \)
       \( 2x + 3y + z = 8 \)
    b) \( [x, y, z] = [6, 11, 1] + t[1, 5, 2] \)
       \( x + 3y + 2z - 1 = 0 \)

8.6 Intersections of Planes
13. Find the line of intersection for these two planes.
    \( 2x + y - 2z = 4 \)
    \( x + 2y - 3z = 8 \)

14. Solve each system of planes.
    a) \( x + y - 3z = 2 \)
       \( 2x - z = 5 \)
       \( 7x + 3y - 11z = 16 \)
    b) \( x + 2y - z = -3 \)
       \( 4x + y - 3z = -3 \)
       \( 2x + y + z = 3 \)
    c) \( x + y + z = 5 \)
       \(-x + y + 2z = -3 \)
       \(2x + 4y + 5z = 0 \)
Chapter 8 Test

For questions 1 and 2, choose the best answer.

1. Which is a vector equation of a line passing through the points (4, 1) and (8, −2)?
   a) \( \mathbf{r} = [4, 1] + t[8, -2] \)
   b) \( \mathbf{r} = [8, -2] + t[4, 1] \)
   c) \( \mathbf{r} = [8, -2] + t[4, -3] \)
   d) \( \mathbf{r} = [4, -3] + u[8, -2] \)

2. Which pair of lines represented by vector equations are coincident?
   a) \( \mathbf{r} = [-2, 5] + s[2, -1] \)
   \( \mathbf{r} = [12, -30] + t[5, -7] \)
   b) \( \mathbf{r} = [4, -1] + s[-3, 5] \)
   \( \mathbf{r} = [-2, 9] + t[-3, 5] \)
   c) \( \mathbf{r} = [0, 0] + s[1, 1] \)
   \( \mathbf{r} = [-1, 1] + t[1, -1] \)
   d) \( \mathbf{r} = [5, 4] + s[2, -4] \)
   \( \mathbf{r} = [9, -8] + t[2, -4] \)

3. A line passes through the point (4, −3) with direction vector \( \mathbf{m} = [1, 5] \).
   a) Determine the parametric equations of the line.
   b) What point on the line corresponds to the parameter value \( t = 2 \)?
   c) Does the line contain the point P(3, −7)?

4. Find a vector equation and the parametric equations of a line through the points A(1, −3, 2) and B(9, 2, 0).

5. Find a vector equation and the parametric equations of a line parallel to the \( y \)-axis and containing the point (1, 3, 5).

6. Write the scalar equation of the line through the point Q(4, −1) with normal \( \mathbf{n} = [3, 5] \).

7. Determine if the lines in each pair intersect. If they intersect, find the intersection point.
   a) \( x = 1 + 3t \)
   \( y = 5t \)
   \( z = 4t - 3 \)
   \( [x, y, z] = [0, -9, -1] + s[-1, 2, -3] \)
   b) \( [x, y, z] = [1, 2, 1] + s[1, -1, -1] \)
   \( [x, y, z] = [0, 2, 5] + s[0, 1, -1] \)

8. Find the distance between these two skew lines.
   \( \ell_1 : [x, y, z] = [1, 3, 7] + t[-1, 1, 2] \)
   \( \ell_2 : [x, y, z] = [4, -2, 1] + s[3, 2, -5] \)

9. Find a vector equation and the parametric equations of the plane that contains the point (3, −5, 1) and is parallel to \( \mathbf{n} = [-5, 2, -5] \).
   \( [x, y, z] = [-5, 2, -5] + t[3, -1, 1] + s[1, 1, 1] \).

10. Find a vector equation of the plane containing the points G(4, 1, −1), H(0, 1, 2), and I(1, 1, −1).

11. Find the scalar equation of the plane containing both the line of intersection of the planes defined by \( 2x - 3y + z - 2 = 0 \) and \( x + 2y - z + 5 = 0 \) and the point P(1, 0, −2).

12. Determine the scalar equation of the plane with a vector equation
   \( [x, y, z] = [3, 0, 2] + t[6, 2, 0] + s[2, 0, -1] \).

13. Determine if the line and the plane intersect. If so, determine the point of intersection.
   a) \( [x, y, z] = [4, 6, 0] + t[-1, 2, 1] \)
   \( 2x - y + 6z + 10 = 0 \)
   b) \( [x, y, z] = [4, -1, 3] + t[3, 3, -4] \)
   \( -5x + 2y - z = -5 \)

14. Determine if the planes in each set intersect. If so, describe how they intersect.
   a) \( x + 2y + 3z = 4 \)
   \( 2x + 4y + 6z - 7 = 0 \)
   \( x + 3y + 2z = 3 = 0 \)
   b) \( x + 2y + 3z = -4 \)
   \( x - y - 3z = 8 \)
   \( x + 5y + 9z = -10 \)
   c) \( 3x + z + 11 = 0 \)
   \( 2x + y + z + 4 = 0 \)
   \( x + y + z - 3 = 0 \)
   d) \( 2x - y + 4z = -7 \)
   \( 3x - 14y + z = -48 \)
   \( x + 2y + 3z = 4 \)

Name: _______________________________  Date: ________________________
Chapter 8 Practice Masters Answers

Prerequisite Skills

1. a) ______ b) ______

2. a) x-intercept: \(\frac{5}{2}\); y-intercept: 5
   b) x-intercept: –4; y-intercept: 20
   c) x-intercept: 6; y-intercept: –3
   d) x-intercept: 6; y-intercept: 3

3. a) ______ b) ______

4. a) (1, –3) b) (4, 3)

5. a) \((4, \frac{-3}{2})\)  b) \((\frac{4}{3}, \frac{-1}{3})\)  c) (11, –2)  d) (2, 7)

6. a) \(y = 3x + 4\)  b) \(y = -\frac{2}{3}x + \frac{16}{3}\)
   c) \(y = \frac{5}{3}x - 4\)  d) \(y = \frac{3}{2}x - 10\)

7. a) no  b) no  c) yes  d) yes  e) no  f) no

8. a) \([-18, 8], [10, 4]\)  b) \([3, 5], [73, 6, 23]\)

9. Answers may vary. a) \([2, -4]; [2, 1]\)
   b) \([10, 14]; [-7, 5]\)  c) \([9, -6, 15]; [-5, 0, 3]\)
   d) \([4, -14, 2]; [1, 0, -2]\)

8.1 Equations of Lines in Two-Space and Three-Space

1. Answers may vary.
   a) \([x, y] = [9, 4] + t[2, -7]\)
   b) \([x, y] = [3, 7] + t[-1, 5]\)
   c) \([x, y, z] = [8, -11, 2] + t[2, -3, 5]\)
   d) \([x, y, z] = [7, -1, 4] + t[-3, 4, 7]\)

2. Answers may vary.
   a) \([x, y] = [2, 3] + t[5, -2]\)
   b) \([x, y] = [5, -1] + t[-7, 8]\)
   c) \([x, y, z] = [4, -3, 1] + t[-2, 0, 6]\)
   d) \([x, y, z] = [5, 6, -1] + t[-5, -9, 3]\)

3. a) No  b) Yes  c) Yes  d) No

4. a) \(x = 7 + 2t, y = 3 + 5t, z = -1 + 8t\)
   b) \(x = -1 + 8t, y = 4 - 9t\)
   c) \(x = 3t, y = 4 - 4t, z = -7 + t\)
   d) \(x = 7 - 2t, y = 5 + t, z = -4 + 3t\)

5. Answers may vary.
   a) \([x, y] = [2, 1] + t[-4, -5]\)
   b) \([x, y] = [8, 3] + t[-9, 0]\)
   c) \([x, y, z] = [4, 7, 1] + t[3, -2, 1]\)
   d) \([x, y, z] = [0, 5, -3] + t[4, -9, 0]\)

6. a) \(5x + 4y - 11 = 0\)  b) \(-7x + 8y + 54 = 0\)
   c) \(-2x + 6y + 10 = 0\)  d) \(7x + z + 9 = 0\)
   e) \(-3x + 4y + z - 13 = 0\)

7. Answers may vary. \([x, y, z] = [7, 8, -3] + t[-9, -5, 8]; x = 7 - 9t, y = 8 - 5t, z = -3 + 8t\)

9. a) Yes  b) No  c) Yes  d) No

10. Answers may vary.
    a) \([x, y] = [2, -7] + t[0, 1]\)
    b) \([x, y] = [-2, 4] + t[3, 4]\)
    c) \([x, y] = [3, 0] + t[-3, 5]\)
    d) \([x, y, z] = [1, -7, 3] + t[-4, -1, 2] + s[-2, 0, 4]\)

8.2 Equations of Planes

1. a) Yes  b) No  c) Yes  d) No

2. a) x-intercept: 6; y-intercept: –4; z-intercept: 2
   b) x-intercept: 56; y-intercept: 8; z-intercept: 7
Chapter 8 Practice Masters Answers

b) 14x + 19y − 3z + 129 = 0
c) y = 8 = 0
d) −30x + 39y + 123z − 99 = 0
e) −7x + 9y + 3z + 61 = 0
f) 5x − 4y + 7z − 70 = 0
g) −3x + 2y + 5z − 11 = 0
h) −8x − 4y − 4z + 28 = 0

8.4 Intersections of Lines in Two-Space and Three-Space

1. a) (2, 3) b) (3, −2) c) (−16, 11) d) (4, 3)
e) (4, 8) f) (−3, 7) g) \( \left( \frac{19}{5}, \frac{7}{5} \right) \) h) (6, 7)

2. a) Yes b) Yes c) No

3. \( 2\sqrt{37}; 2; 10\sqrt{2} \)

4. \( \overline{AC} = [−1, −4] + s[7, −2], \)\n\( \overline{BD} = [1, −3] + t[3, −4]; \) \( \left( \frac{5}{2}, −5 \right) \)

5. a) Infinitely many solutions b) No
c) (−2, −5, −7) d) No e) (8, 2, 3)

6. a) 2.45 b) 5.15 c) 2.73 d) 2.08

8.5 Intersections of Lines and Planes

1. a) Yes b) No

c) Yes d) No

2. a) \( \left( \frac{5}{3}, \frac{4}{3}, \frac{19}{3} \right) \) b) (4, 1, −3) c) \( \left( \frac{39}{5}, \frac{37}{5}, 0 \right) \)
d) (2, 6, 0) e) (4, 8, 3) f) (5, 0, 9)

3. a) 1.70 b) 5.71 c) 2.86 d) 1.08

4. a) 2.5 b) 0.41 c) 4.90 d) 1.36

5. a) 0 b) 2 c) 1.94 d) 7 e) 4 f) 2.12

6. 0.20

8.6 Intersections of Planes

1. Answers may vary.

2. a) \( \left[ \frac{5}{3}, 0, −1 \right] + t\left[ \frac{2}{3}, 1, 0 \right] \)

b) Not possible c) Not possible

2. a) (−1, 2, 2) b) (−10, 13, 1) c) (4, 1, 2)
d) (6, −11, 0) e) (1, 2, 3) f) (4, −2, 3)

3. Answers may vary.

a) \( \left[ \frac{7}{13}, \frac{17}{13}, 0 \right] + t\left[ \frac{1}{13}, \frac{34}{13}, 1 \right] \)

b) \( \left[ \frac{7}{13}, \frac{17}{13}, 0 \right] + t\left[ \frac{1}{13}, \frac{34}{13}, 1 \right] \)
Chapter 8 Practice Masters Answers

2. B
3. a) \( x = 4 + t, y = -3 + 5t \)  b) (6, 7)  c) no
4. Answers may vary.
   \([x, y, z] = [1, -3, 2] + t[8, 5, -2]\)
5. Answers may vary.
   \([x, y, z] = [1, 3, 5] + t[0, 1, 0]\)
6. \(3x + 5y - 7 = 0\)
7. a) \((-2, -5, -7)\)  b) No
8. 0.19
9. Answers may vary.
   \([x, y, z] = [3, -5, 1] + t[3, -1, 1] + s[1, 1, 1]\)
10. Answers may vary.
    \([x, y, z] = [4, 1, -1] + t[-4, 0, 3] + s[-3, 0, 0]\)
11. \(9x - 10y + 3z - 3 = 0\)
12. \(-x + 3y - 2z + 7\)
13. a) \((10, -6, -6)\)  b) \((-8, -13, 19)\)
14. a) Two parallel planes intersected by a third plane  b) Intersect in pairs  c) Intersect at a point  d) Intersect at a line

Chapter 8 Review
1. Answers may vary.
   a) \([x, y] = [4, -5] + t[3, 5]\)
   b) \([x, y, z] = [-6, 2, 1] + t[3, 7, -2]\)
   c) \([x, y, z] = [0, 1, 2] + t[1, 0, 0]\)
   d) \([x, y, z] = [4, -5, 3] + t[-1, -2, -2]\)
2. a) \(5x + y - 17 = 0\)  b) \(3x + 2y + 4 = 0\)
3. \(x = -2 + 3t, y = 3 - 2t, z = 7 + 5t; \) no
4. Answers may vary.
   a) \((2, 4, -8), (3, 8, -6), (6, -1, -6)\)
   b) \((0, 1, 5), (3, 0, 7), (-4, 1, 1)\)
   c) \((0, 0, -12), (-4, 0, 0), (0, 3, 0)\)
5. Answers may vary.
   a) \([x, y, z] = [1, 7, 2] + t[3, -7, 3] + s[0, -5, 1]\)
   b) \([x, y, z] = [2, -3, 4] + t[1, 0, 0] + s[0, 0, 1]\)
6. \(2x - 4y + 3z = 0\)
7. \(-25x - 7y - 5z + 77 = 0\)
8. a) \(-x + y - 1 = 0\)  b) \(z + 1 = 0\)
9. a) One; \((17, -22)\)  b) No solutions
10. a) Yes; \(\left(\frac{3}{2}, \frac{1}{2}, \frac{6}{3}\right)\)  b) No
11. 0.74
12. a) \(\left(\frac{5}{6}, \frac{1}{3}, \frac{16}{3}\right)\)  b) \((4, 1, -3)\)
13. Answers may vary.
    \([x, y, z] = [0, 4, 0] + t[1, 4, 3]\)
14. a) \([x, y, z] = \left[\frac{5}{2}, -\frac{1}{2}, 0\right] + t\left[\frac{1}{2}, -\frac{5}{2}, 1\right]\)
    b) \((1, -1, 2)\)  c) Inconsistent

Practice Test
1. C